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Retailer's Lot-sizing Decisions with Imperfect Quality and Capacity Constraint under Generalized Payments in Three-level Supply Chain

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Abstract.

Every ordered lot may have some fraction of defectives which may vary from process to process and the defective items can be secluded from the good quality lot through a careful inspection process. Thus, an inspection process is obligatory in today's technology driven industry. Besides, an advance payment is used to avoid order cancellations and a credit payment is applied to stimulate sales. Furthermore, this article develops a two-warehouse inventory model that jointly considers the imperfect quality items and hybrid payment schemes for the practical scenario. Mathematical models are developed to depict the above scenario and the aim of this article is to find the optimal lot size to minimize the total cost. Finally, numerical examples are provided to illustrate the results of the proposed model, sensitivity analysis of the inventory model's parameters are done and some managerial insights are given in this article.

1. Introduction

Trade credit is an essential tool for financing growth for many firms. As we all know, various trade credits offered by the sellers motivate the buyers to place larger orders and plan revenue streams accordingly. The number of days for which a credit is given is agreed on by both the sellers allowing the credit and the buyers receiving it, then trade credit gives flexibility to the buyers in making payments. By payment extension date, the buyers essentially could sell the items and use the credit amount to pay back the debt. Condition of permissible delay in payment was not considered prior to 1985 when Goyal [8] in a maiden paper developed an EOQ model under such condition. Lashgari and Taleizadeh [23] developed an inventory control problem for deteriorating items with backordering and financial considerations under two levels of trade credit linked to order

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quantity. Heydari et al. [11] discussed a two-level delay in payments contract for supply chain coordination in the case of credit-dependent demand. On the other hand, the buyers often asked for credit-risk customers to cover a fraction of the purchasing cost as a collateral deposit at the time of placing an order, and then provide a permissible delay on the remaining quantity (i.e. partial trade credit) to reduce default risks as well. Several research papers in this interesting area were published such as Chen and Teng [3], Mahata [28], Tiwari et al. [54], Taleizadeh et al. [52], Mahata and De [29], Maihami et al. [32], Mukherjee and Mahata [34], Mahata et al. [30, 31] and their references.

Naturally, from the seller's perspective, he/she may not be able to collect all of the money from some default buyers when they giving the buyer a credit payment that results in the seller loses interest incomes during the credit period. Therefore, to avoid interest losses, the order cancellation and non-payment risks, we see that the sellers usually asks the buyers to prepay the entire or a fraction of the procurement cost when singing a contract. The prepayment might bring some benefits to the sellers such as they can earn some interest from advance payments, the probability for default risk is zero and there would be no order cancellations. Taleizadeh [46] developed an EOQ model for deteriorating items in a purchasing system with multiple prepayments. Zhang et al. [62] explored an advance-credit payment so that the sellers ask the buyers for a partial advance payment of the purchase amount to avoid the risk of order cancellation. Eck and Engemann [7] explored the role of cash-in-advance financing for export decisions in firms. Lashgari et al. [22] considered partial upstream advanced payment and partial upstream delayed payment in a three-level supply chain. Teng et al. [55] established lot-size policies for deteriorating items when the purchase amount must be prepaid in multiple instalments. Tavakoli and Taleizadeh [51] gave a lot sizing model for deteriorating items with full advance payment from the retailer and conditional discount from the supplier. Lashgari et al. [24] built an EOQ model with upstream partial advance payment and downstream partial credit payment with or without shortages. Wu et al. [61] explored the optimal inventory policies for deteriorating items when the sellers demanded an advance-cash-credit payment type and the buyers in turn offer customers a downstream credit period. Li et al. [27] extended the EOQ model under advance-cash-credit payments to obtain the buyer's optimal selling price and credit period. These topics have been investigated by a large amount of researchers such as Taleizadeh and Nematollahi [48], Taleizadeh [47], Zia and Taleizadeh [63], Diabat et al. [6], Li et al. [25, 26], Taleizadeh [50], Shah et al. [43], Krommyda et al. [18], Tsao et al. [57], Shi et al. [44] and so on.

Based on the above arguments, it is a constant challenge to decide optimal payment type-advance, cash or credit for the sellers to maximize their profit. Again, delayed payments can reduce the buyer's inventory holding cost indirectly, then the sellers encourage the buyers to purchase comparatively more quantity at once by offering a trade credit policy. Herein, the quantity of the items may exceed the buyers own warehouses storage capacity and they must rent another warehouse or rebuild a new warehouse which is located at some distance away from the own warehouse for additional storage. Thereafter, a two-warehouse inventory model was first developed by Hartley [10]. Sarma [41] extended Hartley's model by introducing the transportation cost. Chung and Huang [2] derived an optimal retailer's ordering policies for deteriorating items with limited storage capacity under trade credit financing. Liang and Zhou [19] developed the two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Liao et al. [20, 21] determined an optimal lot size for a two-warehouse system under different assumptions. Panda et al. [38] combined the factors of price, stock and trade credit in a two-warehouse inventory analysis. Many researchers such as Kumar et al. [14], Ranjan and Uthayakumar [40], Tiwari et al. [53], Singh et al. [45], Udayakumar and Geetha [59], Chauhan and Yadav [4] and their references have studied and published many articles in this area.

Finally, product quality is one of the factors that makes firms have competitive and cost-efficient in the long run. Especially, for non-manufacturing firms, a quality inspection must be conducted before selling out the items like electronics items e.g. calculator, trimmers, health parameter measuring equipment in which inspection requires more time and care. Besides, the buyers will perform an inspection process to assess the defectives items in a lot when they receive the ordered lot. The process may result in the separation of items with differing quality. Such type of imperfect production was first studied by Rosenblett and Lee [39]. Salameh and Jaber [42] assumed that the defective item could be sold at a discounted price in a single batch by the end of the 100% screening process and found that the economic lot size quantity tends to increase as the average percentage of imperfect quality items increase. Goyal and Cárdenas-Barrón [9] made some modifications in the model of Salameh and Jaber [42] developed a near-optimal approximation for the lot size. Jaggi et al. [12] established a buyer's optimal ordering and pricing decisions for deteriorating items in a two-warehouse environment. Wang et al. [60] developed an EOQ model for imperfect quality items with partial backorders and screening constraint. Pasandideh et al. [35] presented a multi-product single machine EPQ model for an imperfect production system under warehouse construction cost. Jaggi et al. [13] developed a two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments. Taleizadeh et al. [49] developed an imperfect EPQ model with up-stream trade credit periods linked to raw material-order quantity and downstream trade credit periods. Khanna et al. [15] developed a model for deteriorating imperfect quality items with allowable shortages and permissible delay in payments. Zhou et al. [64] found a synergy economic order quantity model, in which the concepts of imperfect quality, inspection error and shortages with trade credit are considered. Alamri et al. [1] developed an inventory control model for imperfect quality items. Datta [5] proposes a production-inventory model with defective items that incorporates additional invest-ment opportunity on quality improvement for reducing the proportion of defective products. Pal and Mahapatra [36] developed an inventory model with imperfect products for a three-level supply chain, and three different ways of dealing with defective products were investigated in their model. Other papers related to two warehouse inventory model or imperfect quality items are Palanivel and Uthayakumar [37], Khanna et al. [16], Tsao et al. [56, 58], Mandal and Giri [33] and Kazemi et al. [17].

Considering the importance of above said facts, this article proposes a two-warehouse inventory model that considers imperfect quality items under the conditions of that the supplier has the power of controlling or influencing another members decisions. Thereafter, we assume the supplier asks the retailer to prepay a fraction of the purchasing cost when signing a contract to buy items, to pay another fraction of the purchasing cost in cash upon receiving the order quantity and to receive a short-term interest-free credit term on the remaining purchasing. In turns, the use of a downstream partial credit with credit-risk customers is considered in this paper. By applying the functional properties of the total cost, it is proved that there exists a unique optimal solution for each case. Numerical examples are provided to illustrate the results of the proposed model and provide some managerial insights.

2. Notation and Assumptions

Assumptions

The mathematical model of the two-warehouse system is based on the following assumptions:

(1) The demand rate is known with certainly and is uniform.

(2) The time horizon of the inventory system is infinite.

(3) Replenishment is instantaneous.

(4) Shortages are not allowed.

(5) The owned warehouse (OW) has the finite capability of w units and the rented warehouse (RW) has infinite capability.

(6) Both inspection and demand proceeds simultaneously, but the inspection rate is assumed to be greater than demand rate, $x > D$.

(7) The defective items exist in lot size y.

(8) The holding cost is higher in RW than in OW. Based on this assumption, it is economical to store in OW first and after it is filled, RW is used. But while emptying RW is used first and OW next.

(9) Transportation cost for delivering from RW to OW is implicitly included in holding cost of the RW.

(10) The payments scheme for the retailer consists of three parts:

  (i) the advance payment: the supplier demands the retailer prepay α percentage of the purchasing cost at time - L years prior to the time of delivery.

  (ii) the cash payment: the supplier demands the retailer pay another β percentage of the purchasing cost at the time of delivery.

  (iii) the credit payment: the supplier offers a credit period of S years on the remaining τ percentage of the purchasing cost.

(11) The retailer offers a credit period of R years to his/her customers. The customers have to pay $1-\rho$ percentage of the purchasing cost at the time of placing an order and rest of the payment would be made at R.

Notation

The following nomenclature is used throughout this paper.

D	annual demand rate known and constant (unit/time unit).
K	cost of placing an order of the retailer.
x	inspection rate per unit per unit time.
h_w	the holding cost per unit per year in dollars excluding interest charge in OW.
h_R	the holding cost per unit per year in dollars excluding interest charge in RW, where $h_w \leq h_R$.
d	inspection cost per unit item (\$/unit).
w	the maximum allowable number of items in the owned warehouse.
r	percentage of defective items (per unit), where $0 \leq r < 1$.
p	the selling price per unit item of the retailer.
v	the purchasing cost per unit item of the retailer, where $p > v$.
t_r	inspection time of the RW (years).
t_w	inspection time of the OW (years).
t_o	the time at which inventory level reaches zero in RW.
S	the upstream credit period by the supplier to the retailer.
R	the downstream credit period by the retailer to the customers.
L	the length of time during which the prepayment are made, $L > 0$.
I_c	interest charged per dollar per year.
I_e	interest earned per dollar per year.
ρ	the fraction of the sales revenue offered a permissible delay by the retailer to customers $0 \leq \rho \leq 1$.
α	the fraction of purchasing cost to be paid in advance, $0 \leq \alpha \leq 1$.
β	the fraction of purchasing cost to be paid at the time delivery, $0 \leq \beta \leq 1$.
τ	the fraction of procurement cost granted a permissible delay from the supplier to the retailer, $0 \leq \tau \leq 1$, $\alpha + \beta + \tau = 1$.
y	retailer's order size per cycle (units).
T	retailer's replenishment cycle time (unit of time).
T^*	optimal replenishment cycle time (unit of time).
$TC(T)$	cost per unit of time of the retailer.

3. Mathematical Formulation of the Proposed Inventory Model

When the lot comes in the inventory system, then the w units of the item are stored in OW, and the remaining units $(y - w)$ of the item are placed in RW. Initially, for the reasons of weak process control, low skilled labor, low-quantity raw materials and improper handling during transportation, the production process cannot produce 100% good quality items which results in an inspection process for the produced items must be carried on at rate x when the lot arrives in the inventory. Now, let $N_{rw}(r, y)$ and $N_{ow}(r, y)$ be the number of non-defective items in an order of size y for RW and OW, respectively, which are

$$N_{rw}(r,y) = (y-w)(1-r), \tag{3.1}$$

and

$$N_{ow}(r,y) = w(1-r). \tag{3.2}$$

Next, the inspection of items is done in OW and RW at the time t_w and t_r, respectively. To avoid shortages, the minimum number of standard quality items, $N_{rw}(r,y)$ and $N_{ow}(r,y)$ is up to the customers demand during the inspection period, that is

$$N_{rw}(r,y) \geq Dt_r, \tag{3.3}$$

and

$$N_{ow}(r,y) \geq Dt_w. \tag{3.4}$$

By substituting eqs. (3.1) and (3.2) into eqs. (3.3) and (3.4), next, replacing $t_r = \frac{y-w}{x}$ and $t_w = \frac{w}{x}$ into eqs. (3.3) and (3.4), so we have

$$r \leq 1 - \frac{D}{x}. \tag{3.5}$$

As above-calculation in inventory level, the inventory-related costs per unit of time include the following elements:

(1) The ordering cost is $\frac{K}{T}$.

(2) The purchasing cost per year is $\frac{vy}{T} = \frac{vD}{1-r}$.

(3) The inspection cost is $\frac{dy}{T} = \frac{dD}{1-r}$.

(4) The stock holding cost : Based on the values of time value t_w and t_r, there are two cases to be studied.

Case 1: Suppose that $t_w \leq t_o$

The stock holding cost in RW is

$$\frac{h_R}{T}\Big[\frac{t_o(y-w)(1-r)}{2}+r(y-w)t_r\Big] = h_R\Big\{\Big[D^2T-2Dw(1-r)+\frac{w^2(1-r)^2}{T}\Big]\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big]\Big\}.$$

The stock holding cost in OW is

$$\frac{h_w}{T}\Big[wt_w + w(t_o-t_w)(1-r) + \frac{w(T-t_o)(1-r)}{2}\Big] = h_w\Big[\frac{rw^2}{xT} - \frac{w^2(1-r)^2}{2DT} + w(1-r)\Big].$$

Case 2: Suppose $t_w > t_o$

The stock holding cost in RW is

$$\frac{h_R}{T}\Big[\frac{t_o(y-w)(1-r)}{2}+r(y-w)t_r\Big] = h_R\Big\{\Big[D^2T-2Dw(1-r)+\frac{w^2(1-r)^2}{T}\Big]\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big]\Big\}.$$

The stock holding cost in OW is

$$\frac{h_w}{T}\Big[wt_o + rw(t_w - t_o) + \frac{w(T-t_o)(1-r)}{2}\Big] = h_w\Big[\frac{rw^2}{xT} - \frac{w^2(1-r)^2}{2DT} + w(1-r)\Big].$$

It is easy to see that the stock holding cost in above cases are precisely the same.

(5) The retailer's interest charged for both advance-cash payments is given below:

$$\frac{vDI_c}{T}\Big[T\Big(\int_{-L}^{R}\alpha dt + \int_0^R \beta dt\Big) + (\alpha+\beta)\int_R^{T+R}(T+R-t)dt\Big]$$
$$= vDI_c\Big\{[\alpha(R+L)+\beta R] + \frac{\alpha+\beta}{2}T\Big\}.$$

(6) The retailer's interest earned and interest charged for the credit payment are calculated as follows.

Case 1: $R \le S$ and $S \le T+R$

The retailer accumulates revenue and earns interest from that the cash payment starting from time 0 to S and the credit payment starting from time R to S. On the other side, the retailer must finance all items sold after time S for the cast payment, and all items sold after time $S-R$ for the credit payment. Under this situation, the interest earned and the interest charged are as follows:

$$IE = \frac{\tau p DI_e}{T}\Big[\rho\int_R^S (t-R)dt + (1-\rho)\int_0^S t dt\Big] = \frac{\tau p DI_e}{2T}[\rho(S-R)^2 + (1-\rho)S^2],$$

and

$$IC = \frac{\tau v DI_c}{T}\Big[\rho\int_S^{T+R}(T+R-t)dt + (1-\rho)\int_S^T (T-t)dt\Big]$$
$$= \frac{\tau v DI_c}{2T}[\rho(T+R-S)^2 + (1-\rho)(T-S)^2].$$

Case 2: $R \le S$ and $S \ge T+R$

The retailer receives all payments before the credit period S, so there is no interest charged. Conversely, the interest earned is

$$IE = \tau p DI_e\Big[\rho\int_R^{T+R}(t-R)dt + \rho\int_{T+R}^S T dt + (1-\rho)\int_0^T t dt + (1-\rho)\int_T^S T dt\Big]\Big/T$$
$$= \frac{\tau p DI_e}{2T}\Big\{\rho[2T(S-R) - T^2] + (1-\rho)T(2S-T)\Big\}.$$

Case 3: $R > S$

In this case, there is no interest earned from the credit payment. Conversely, the interest charged for credit payment is as follows:

$$IC = \tau v DI_c\Big\{\rho\int_S^R T dt + \rho\Big[\int_R^{T+R}(T+R-t)dt\Big] + (1-\rho)\Big[\int_S^T (T-t)dt\Big]\Big\}\Big/T$$

$$= \frac{\tau v D I_c}{2T}\Big\{\rho[2T(R-S)+T^2]+(1-\rho)(T-S)^2\Big\}.$$

Based on the above descriptions, the total cost per unit of time is given by

$TC(T) =$ ordering cost + purchasing cost + stock holding cost in OW + stock holding cost in RW + inspection cost + interest charged − interest earned.

Thus, we have

$$TC(T) = \begin{cases} TC_1(T) \text{ if } R \le S, & (3.6.a) \\ TC_2(T) \text{ if } R > S, & (3.6.b) \end{cases}$$

where

$$TC_1(T) = \begin{cases} TC_{11}(T) \text{ if } S \le T+R, & (3.7.a) \\ TC_{12}(T) \text{ if } S > T+R, & (3.7.b) \end{cases}$$

$$TC_2(T) = TC_{21}(T) \text{ if } S \le T, \tag{3.8}$$

where

$$\begin{aligned} TC_{11}(T) =& \frac{K}{T}+h_R\Big[D^2T-2Dw(1-r)+\frac{w^2(1-r)^2}{T}\Big]\Big(\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big) \\ &+h_w\Big[\frac{rw^2}{xT}-\frac{w^2(1-r)^2}{2DT}+w(1-r)\Big]+\frac{Dd}{1-r}+vDI_c\Big[\alpha(R+L)+\beta R+\frac{\alpha+\beta}{2}T\Big] \\ &+\frac{\tau vDI_c}{2T}[\rho(T+R-S)^2+(1-\rho)(T-S)^2]-\frac{\tau pDI_e}{2T}[\rho(S-R)^2+(1-\rho)S^2] \quad (3.9) \\ TC_{12}(T) =& \frac{K}{T}+h_R\Big[D^2T-2Dw(1-r)+\frac{w^2(1-r)^2}{T}\Big]\Big(\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big) \\ &+h_w\Big[\frac{rw^2}{xT}-\frac{w^2(1-r)^2}{2DT}+w(1-r)\Big]+\frac{Dd}{1-r}+vDI_c\Big[\alpha(R+L)+\beta R+\frac{\alpha+\beta}{2}T\Big] \\ &+\frac{\tau pDI_e}{2T}\{\rho[2T(S-R)-T^2]+(1-\rho)T(2S-T)\} \quad (3.10) \end{aligned}$$

and

$$\begin{aligned} TC_{21}(T) =& \frac{K}{T}+h_R\Big[D^2T-2Dw(1-r)+\frac{w^2(1-r)^2}{T}\Big]\Big(\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big) \\ &+h_w\Big[\frac{rw^2}{xT}-\frac{w^2(1-r^2)}{2DT}+w\Big]+\frac{Dd}{1-r}+vDI_c\Big[\alpha(R+L)+\beta R+\frac{\alpha+\beta}{2}T\Big] \\ &+\frac{\tau vDI_c}{2T}\{\rho[2T(R-S)-T^2]+(1-\rho)T(T-S)^2\}. \quad (3.11) \end{aligned}$$

It is observed that $TC_{11}(S-R) \ge TC_{12}(S-R)$ if $vI_c \ge pI_e$ and $TC_{11}(S-R) \le TC_{12}(S-R)$ if $vI_c \le pI_e$, so we get that $TC_1(T)$ is well-defined and continuous except at $T = S-R$. Later, by taking the first and the second derivatives of $TC_{11}(T)$, $TC_{12}(T)$ and $TC_{21}(T)$ with respect to T, we have

$$TC'_{11}(T) = \frac{1}{T^2}\Big\{-K+h_R\Big(\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big)[D^2T^2-w^2(1-r)^2]+h_w w^2\Big[\frac{(1-r)^2}{2D}-\frac{r}{x}\Big]$$

$$+\frac{vDI_c}{2}(\alpha+\beta)T^2+\frac{\tau vDI_c}{2}\{\rho[T^2-(R-S)^2]+(1-\rho)(T^2-S^2)\}$$
$$+\frac{\tau pDI_e}{2}[\rho(S-R)^2+(1-\rho)S^2]\Big\} \tag{3.12}$$

$$TC''_{11}(T)=\frac{1}{T^3}\Big\{2K+2h_Rw^2(1-r)^2\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big]+h_ww^2\Big[\frac{2r}{x}-\frac{(1-r)^2}{D}\Big]$$
$$+\tau vDI_c[(1-\rho)S^2+\rho(R-S)^2]-\tau pDI_e[\rho(S-R)^2+(1-\rho)S^2]\Big\}$$
$$>\frac{1}{T^3}\Big\{2K+h_ww^2\Big(\frac{4r}{x}\Big)+\tau D[(1-\rho)S^2+\rho(R-S)^2](vI_c-pI_e)\Big\} \tag{3.13}$$

$$TC'_{12}(T)=\frac{1}{T^2}\Big\{-K+h_R\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big][D^2T^2-w^2(1-r)^2]+h_ww^2\Big[\frac{(1-r)^2}{2D}-\frac{r}{x}\Big]$$
$$+\frac{vDI_c}{2}(\alpha+\beta)T^2-\frac{\tau pDI_e}{2}T^2\Big\} \tag{3.14}$$

$$TC''_{12}(T)=\frac{1}{T^3}\Big\{2K+2h_Rw^2(1-r)^2\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big]+h_ww^2\Big[\frac{2r}{x}-\frac{(1-r)^2}{D}\Big]\Big\}$$
$$>\frac{1}{T^3}\Big\{2K+h_ww^2\Big(\frac{4r}{x}\Big)\Big\}>0 \tag{3.15}$$

$$TC'_{21}(T)=\frac{1}{T^2}\Big\{-K+h_R\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big][D^2T^2-w^2(1-r)^2]+h_ww^2\Big[\frac{(1-r)^2}{2D}-\frac{r}{x}\Big]$$
$$+\frac{vDI_c}{2}(\alpha+\beta)T^2+\frac{\tau vDI_c}{2}[\rho T^2+(1-\rho)(T^2-S^2)]\Big\} \tag{3.16}$$

and

$$TC''_{21}(T)=\frac{1}{T^3}\Big\{2K+2h_Rw^2(1-r)^2\Big[\frac{1}{2D}+\frac{r}{x(1-r)^2}\Big]+h_ww^2\Big[\frac{2r}{x}-\frac{(1-r)^2}{D}\Big]$$
$$+\tau vDI_c(1-\rho)S^2\Big\}$$
$$>\frac{1}{T^3}\Big\{2K+h_ww^2\Big(\frac{4r}{x}\Big)+\tau vDI_c(1-\rho)S^2\Big\}>0. \tag{3.17}$$

Clearly, eqs. (3.15) and (3.17) imply that $TC_{12}(T)$ and $TC_{21}(T)$ are convex on $T>0$. Eq. (3.13) imply that $TC_{11}(T)$ is convex on $T>0$ if $2K>\tau pDI_e[\rho(S-R)^2+(1-\rho)S^2]$. Otherwise, $TC_{11}(T)$ is increasing on $T>0$. Here, as T approaches to infinite, $TC'_{1i}(T)$ approaches to infinite $(i=1,2)$. Let T_i^* denote the root of $TC'_{1i}(T)=0$ $(i=1,2)$. By the convexity of $TC_{1i}(T)$ $(i=1,2)$, we have

$$TC'_{1i}(T)\begin{cases}<0 & \text{if} \quad T<T_i^*\\ =0 & \text{if} \quad T=T_i^*\\ >0 & \text{if} \quad T>T_i^*\end{cases} \qquad (i=1,2). \tag{3.18}$$

It is showed that the obtained optimal solution exists and it is unique by the Intermediate Value Theorem. Given above, the retailer's objective here is to find the optimal replenishment cycle such that the total cost $TC(T)$ per unit of time in eqs. (3.6a, 3.6b) is minimized. For notational convenience, $\delta=2K-\tau pDI_e[\rho(S-R)^2+(1-\rho)S^2]$, we have

$$TC'_{11}(S-R) = \frac{1}{(S-R)^2}\Delta_1, \tag{3.19}$$

and

$$TC'_{12}(S-R) = \frac{1}{(S-R)^2}\Delta_2, \tag{3.20}$$

where

$$\begin{aligned}\Delta_1 =& -K + h_R\Big[\frac{1}{2D} + \frac{r}{x(1-r)^2}\Big][D^2(S-R)^2 - w^2(1-r)^2] + h_w w^2\Big[\frac{(1-r)^2}{2D} - \frac{r}{x}\Big]\\ &+ \frac{vDI_c}{2}(\alpha+\beta)(S-R)^2 + \frac{\tau vDI_c}{2}(1-\rho)[(S-R)^2 - S^2]\\ &+ \frac{\tau pDI_e}{2}[\rho(S-R)^2 + (1-\rho)S^2]\end{aligned} \tag{3.21}$$

and

$$\begin{aligned}\Delta_2 =& -K + h_R\Big[\frac{1}{2D} + \frac{r}{x(1-r)^2}\Big][D^2(S-R)^2 - w^2(1-r)^2] + h_w w^2\Big[\frac{(1-r)^2}{2D} - \frac{r}{x}\Big]\\ &+ \frac{vDI_c}{2}(\alpha+\beta)(S-R)^2 + \frac{\tau pDI_e}{2}(S-R)^2.\end{aligned} \tag{3.22}$$

Herein, we have $\Delta_1 \geq \Delta_2$ if $pI_e \geq vI_c$. Otherwise, $\Delta_1 \leq \Delta_2$. Based on the aforementioned arguments, effective decision rules are developed to find the optimal replenishment cycle for the retailer.

Theorem 1. *Suppose that $\delta > 0$, then the results are as follows :*

(1) *If $pI_e \geq vI_c$,*

- (1-1) $\Delta_1 \geq 0$ *and* $\Delta_2 \geq 0$, *then* $T^* = T_1^* = S-R$ *or* T_2^* *associated with the least cost.*
- (1-2) $\Delta_1 \geq 0$ *and* $\Delta_2 < 0$, *then* $T^* = T_1^* = S-R$.
- (1-3) $\Delta_1 < 0$ *and* $\Delta_2 < 0$, *then* $T^* = T_1^*$.

(2) *If $pI_e < vI_c$,*

- (2-1) $\Delta_1 \geq 0$ *and* $\Delta_2 \geq 0$, *then* $T^* = T_2^*$.
- (2-2) $\Delta_1 < 0$ *and* $\Delta_2 \geq 0$, *then* $T^* = T_1^*$ *or* T_2^* *associated with the least cost.*
- (2-3) $\Delta_1 < 0$ *and* $\Delta_2 < 0$, *then* $T^* = T_1^*$. *or* $T_2^* = S-R$. *associated with the least cost.*

Proof. It can be proved from the above arguments.

Likewise, when $\delta \leq 0$, eq.(3.12) imply that $TC_{11}(T)$ is increasing on $(0,\infty)$ and eq.(3.21) imply that $\Delta_1 > 0$. It is easy to verify the following results.

Theorem 2. *Suppose that $\delta \leq 0$, then the results are as follows:*

(1) *If $pI_e \geq vI_c$,*

(1-1) $\Delta_1 \geq 0$ *and* $\Delta_2 \geq 0$*, then* $T^* = T_1^* = S - R$ *or* T_2^* *associated with the least cost.*

(1-2) $\Delta_1 \geq 0$ *and* $\Delta_2 < 0$*, then* $T^* = T_1^* = S - R$*.*

(2) *If* $pI_e < vI_c$*,* $\Delta_1 \geq 0$ *and* $\Delta_2 \geq 0$*, then* $T^* = T_2^*$*.*

Proof. It can be proved from the above arguments.

Finally, in case of $R \geq S$, eq. (3.17) yield $TC_{21}(T)$ is convex on $(0, \infty)$, likewise, let T_{21}^* denote the root of $TC_{21}'(T) = 0$. By the convexity of $TC_{21}(T)$, we have $T^* = T_{21}^*$.

4. Numerical Examples

To provide a better understanding of the model and the theoretical results, some numerical examples are presented in this section. Examples 1-4 are for the case of $R \leq S$.

Example 1. When $\delta > 0$ and $pI_e \geq vI_c$, we assume an inventory system with the following parameters: $\alpha = 0.4$, $\beta = 0.3$, $\tau = 0.3$, $\rho = 0.3$, $r = 0.2$, the demand rate of a certain product is $D = 500$ units/year, the capacity limit in OW is $w = 50$ units, the ordering cost is $K = \$15$ per order, the purchasing cost is $v = \$10$/unit, the selling price is $p = \$25$/unit/year, the inspection cost is $d = \$2$/unit/year, the inspection rate for imperfect quality items is $x = 1000$ units/year, the holding cost in OW is $h_w = 0.1$/unit/year, the holding cost in RW is $h_R = \$0.3$/unit/year, the lead time is $L = 0.2$ year, the downstream credit period $R = 0.1$ year, upstream credit period $S = 0.3$ year, interest charged $I_c = \$0.15/\$/\text{year}$,, and interest rate earned $I_e = \$0.1/\$/\text{year}$. The parameters are as follows: $\alpha = 0.4$, $\beta = 0.3$, $\tau = 0.3$, $\rho = 0.3$, $r = 0.2$, $x = 1000$, $w = 50$, $K = \$15$, $v = \$10$, $p = \$25$, $h_w = \$0.1$, $h_R = \$0.3$, $d = \$2$, $L = 0.2$, $R = 0.1$, $S = 0.3$, $I_c = \$0.15$ and $I_e = \$0.1$. The optimal results of T^* and $TC(T^*)$ are shown in Table 1.

Table 1: Outcomes for the examples that illustrate Theorem 1.

Theorem 1: $\delta > 0$ and $pI_e \geq vI_c$.				
D	Δ_1	Δ_2	T^*	$TC(T^*)$
400	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
300	≥ 0	< 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 892.2167$
200	< 0	< 0	$T_1^* = 0.27$	$TC_1(T_1^*) = 613.9049$
Theorem 1: $\delta > 0$ and $pI_e < vI_c$.				
D	Δ_1	Δ_2	T^*	$TC(T^*)$
500	≥ 0	≥ 0	$T_2^* = 0.1710$	$TC_2(T_2^*) = 1456.9$
400	< 0	≥ 0	$T_2^* = 0.1990$	$TC_2(T_2^*) = 1176.2$
300	< 0	< 0	$T_1^* = 0.2400$	$TC_1(T_1^*) = 894.4019$

Example 2. When $\delta > 0$ and $pI_e < vI_c$, the parameters are the same as Example 1 expect $v=$ \$8 and $p=$ \$10. The optimal results of T^* and $TC(T)^*$ are shown in Table 1.

Example 3. When $\delta \leq 0$ and $pI_e \geq vI_c$, the parameters are the same as Example 1 expect $K = \$2$, $v = \$5$, and $p = \$15$. The optimal results of T^* and $TC(T^*)$ are shown in Table 2.

Table 2: Outcomes for the examples that illustrate Theorem 2.

Theorem 2: $\delta \leq 0$ and $pI_e \geq vI_c$.				
D	Δ_1	Δ_2	T^*	$TC(T^*)$
170	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S-R) = 453.8503$
150	≥ 0	< 0	$T_1^* = S - R = 0.2$	$TC_1(S-R) = 402.4865$
Theorem 2: $\delta \leq 0$ and $pI_e < vI_c$.				
D	Δ_1	Δ_2	T^*	$TC(T^*)$
500	≥ 0	≥ 0	$T_2^* = 0.1110$	$TC_2(T_2^*) = 1385.9$

Example 4. When $\delta \leq 0$ and $pI_e < vI_c$, the parameters are the same as Example 1 expect $K = \$5$, $v = \$8$, and $p = \$10$. The optimal results of T^* and $TC(T^*)$ are shown in Table 2.

Example 5. The case of $R > S$.

Suppose that the parameters of example 1 are changed to $D = 500$, $v = 8$, $p = 10$, $S = 0.12$, and $R = 0.15$, then the optimal solutions are $T_{21}^* = 0.0830$, and the minimum cost is $TC(T^*) = TC(TC_{21}^*) = 1400.3$.

5. Sensitivity Analysis

In this section, we change the value of one parameter from -50% to 50% at a time and keeping same values of the rest parameters for Example 1. The sensitivity tables are given in Tables 3 to 12. After that, we investigate the effect of various important parameters on the optimal solution.

The sensitivity analysis reveals that:

(1) From Table 3, as the parameter D increases, the total cost increases but the length of replenishment cycle decreases.

(2) From Table 4, as the value of own warehouse capacity increases, the total cost decreases. That is, the capacity of own warehouse has negative effects on the total cost.

(3) From Table 5, when the value of v increases, the total cost increases but the length of the replenishment cycle decreases.

(4) From Table 6, as p increases, the length of replenishment cycle increases but the total cost decreases. That is, the selling price has negative effects on the total cost.

Table 3: Impact of the value of D on the optimal solution.

D	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
600	Theorem 2-1	≥ 0	≥ 0	$T_2^* = 0.1340$	$TC_2(T_2^*) = 1719.00$
400	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
200	Theorem 1-1	< 0	< 0	$T_1^* = 0.27$	$TC_1(T_1^*) = 613.9049$

Table 4: Impact of the value of w on the optimal solution.

w	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
75	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1165.90$
50	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.90$
25	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1650$	$TC_2(T_2^*) = 1178.60$

Table 5: Impact of the value of v on the optimal solution.

v	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
15	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1610$	$TC_2(T_2^*) = 1232.7$
10	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
5	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1650$	$TC_2(T_2^*) = 1102.3$

Table 6: Impact of the value of p on the optimal solution.

p	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
37.5	Theorem 2-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1141.8$
25	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
12.5	Theorem 1-2	≥ 0	≥ 0	$T_2^* = 0.187$	$TC_2(T_2^*) = 1197.1$

Table 7: Impact of the value of S on the optimal solution.

S	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.45	Theorem 2-1	≥ 0	≥ 0	$T_2^* = 0.1750$	$TC_2(T_2^*) = 1125.2$
0.30	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.15	Theorem 1-2	< 0	< 0	$T_1^* = 0.1800$	$TC_1(T_1^*) = 1211.7$

Table 8: Impact of the value of R on the optimal solution.

R	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.15	Theorem 1-1	< 0	< 0	$T_1^* = 0.1600$	$TC_1(T_1^*) = 1191.3$
0.10	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.05	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1750$	$TC_2(T_2^*) = 1144.7$

Table 9: Impact of the value of I_e on the optimal solution.

I_e	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.15	Theorem 2-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1141.8$
0.10	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.05	Theorem 1-2	≥ 0	≥ 0	$T_2^* = 0.1870$	$TC_2(T_2^*) = 1197.1$

Table 10: Impact of the value of I_c on the optimal solution.

I_c	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.225	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1610$	$TC_1(S - R) = 1232.7$
0.150	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.075	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_2(T_2^*) = 1102.3$

Table 11: Impact of the value of ρ on the optimal solution.

ρ	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.45	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1750$	$TC_2(T_2^*) = 1174.7$
0.30	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.15	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_2(T_2^*) = 1165.0$

Table 12: Impact of the value of r on the optimal solution.

r	Theorem	Δ_1	Δ_2	T^*	$TC(T^*)$
0.3	Theorem 1-1	≥ 0	≥ 0	$T_2^* = 0.1600$	$TC_2(T_2^*) = 1322.1$
0.2	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_1(S - R) = 1169.9$
0.1	Theorem 1-1	≥ 0	≥ 0	$T_1^* = S - R = 0.2$	$TC_2(T_2^*) = 1051.8$

(5) From Table 7, it is obvious that a higher permissible delay period S results in a lower value for the total cost. That is, if the trade credit increases for the retailer, then he/she gets a lower cost from the permissible delay.

(6) From Table 8, the value of the total cost increases as R increases. It is observed that as the length of the credit period proposed to the customer increases, the retailer has positive effects on the total cost.

(7) From Table 9, as I_e increases, the length of replenishment cycle increases, but the total cost decreases. That is, the increase in the interest-earning rates has a direct impact on the cost resulting in lower cost at higher rates.

(8) From Table 10, as the value of I_c increases, the total cost increases but the length of replenishment cycle decreases.

(9) From Table 11, as the value of ρ increases, the total cost increases but the length of replenishment cycle decreases.

(10) From Table 12, as the imperfect portion increases, this increases the total cost.

Managerial implications

It is observed from above tables, with the increase in the value of own warehouse capacity, the total cost decreases since the inventory holding cost in OW is lower than that in RW resulting in lower cost at higher OW capacity. Additionally, from tables 10 and 11, it is showed that the positive impact of the higher value of interest payable rates on costs since it is a direct addition to the retailer's cost and a higher fraction of the sales revenue offered trade credit by the retailer to customers resulting in a higher value for the total cost. Finally, it implies that an increase in the percentage of imperfect items causes a decrease in the total cost due to the inspection cost increases for the retailer, so he/she must investigate the origin of the received lots, and carefully select suppliers for buying.

6. Conclusion

This article has investigated an EOQ model for imperfect quality items with capacity constraint in a three-level supplier-retailer-customer chain in which the payment policies between them are part of the supply chain financing strategy that affect the lot-sizing decisions of each party. Furthermore, we consider the customer would like to pay some portion of the purchase amount in the future days (i.e., a cash-credit payment), in turns, the supplier would like to request a good-faith deposit from the retailer to avoid the order cancellation (i.e., an advance payment), so he/she demands the buyer an advance-cash-credit scheme. Besides, when the order quantity becomes higher than the storage capacity in own warehouse then the system involves two warehouses model.

Again, in real life, the non-manufacturing firms must conduct a quality inspection before selling out the items in the market because the inspection process becomes crucial in ensuring quality to customers in a competitive market. Under above situations, we reveal several theoretical results which are given as Theorems 1-2 to determine the optimal replenishment cycle for the retailer to minimum the total cost under various conditions. Numerical examples are presented to illustrate the inventory models, sensitivity analysis of the model's parameters are done and some managerial insights are given.

The aforementioned results can provide guidance on how to adjust the optimal replenishment cycle for the retailer when different parameters change. It is useful and easy to implement in any firm by practitioners. Finally, it has showed that the increases in the percentage of the imperfect items causes an increase in the total cost, so the retailer must select suppliers for buying carefully.

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