International Journal of
Information
and
Management
Sciences

http://ijims.ms.tku.edu.tw/main.php

International Journal of
Information and Management Sciences

33 (2022), 35-54. DOI:10.6186/IJIMS.202203_33(1).0003

Correlation Coefficient Based Extended VIKOR Approach Under Intuitionistic Fuzzy Environment

Rakhi Gupta and Satish Kumar
Deemed To Be University

Keywords

Renyi's
Intuitionistic fuzzy set (IFSs)
MCDM (Multi-criteria Decision Making)
Intuitionistic fuzzy numbers (IFNs)
VIKOR(Vlsekriterijumska Optimizacija i Kompromisno Resenje)
Shannon's

Abstract.

In this paper, the mathematical modeling of generalization of Shannon entropy has been introduced. Thereafter, we proposed an entropy measure for intuitionistic Fuzzy Set and also examine their properties. We have incorporated a term that is Correlation coefficient and using this term Extended VIKOR (Vlsekriterijumska OPtimizacija i KOmpromisno Resenje) method is employed for determining the best alternative instead of distance measure. Thus, the key innovation is to combine the traditional VIKOR model with intuitionistic fuzzy set based on correlation coefficient method. Further, criteria weight is to be determined for partially known and unknown cases using Extended Vikor model. To implement the application of proposed entropy in MCDM (Multi-criteria Decision Making) problems we have taken numerical example of supplier selection. Moreover, we have obtained the ranking sequence using Extended Vikor method. In addition, we have also provided sensitive analysis for different values of weight to show the reliability of proposed measure. Finally, comparison has been made with different entropy measures to show the compatibility of proposed measure with existing one.

1. Introduction

To deal with ambiguities, fuzzy set was introduced by Zadeh [47]. He had extended classical set theory by assigning membership degree (μ). Initially, probability was the only measure to determined the ambiguities. But for that the terms should be expressed as exact number. The ambiguous terms such as large, very large and very very large can be expressed by Fuzzy Set (FS). Thus, to deal with ambiguity FS is more useful than traditional approaches. Due to the development of FS in different applications it become the most useful research topic among different researchers. Distinct generalizations have been introduced by different researchers (Joshi and Kumar [18], Joshi [22], Arya and Kumar [37], Joshi and Kumar [19]). After some time, intuitionistic fuzzy set was introduced by Atanassov [3] which is further extension of FSs. He had assigned membership (μ) and non-membership (ν) degree and also added an another factor known as intuitionistic index (π). Therefore, IFSs is more adaptable to deal with uncertainty. Let us take an example:- A company want to buy a product and there are two vendors (A and B) in competition. Some of the members are in favor of vendor A, some are against the vendor A while some of the members are confused. In these type of situation intuitionistic fuzzy set is more useful. Thus, it becomes more adaptable to real life problems.

Different researchers had used the concept of IFS and give their entropies. Bhandari and Pal [5] was the first one to give the generalized entropy. Liu and Ren [24] suggested cosine function based intuitionistic fuzzy entropy. Some of the researchers (Mao and Yao [26], Xiong et al. [41] and Mishra [28]) introduced their entropy in Logarithmic function while parameter based entropy was suggested by Joshi and Kumar [21]. To select the best alternative from set of multiple decision alternatives, MCDM techniques are more helpful. Therefore, IFS can be implemented in most of MCDM problems. For solving these type of problems, different ideologis suggested by distinct researchers such as VIKOR (VIsekriterijumska Optimizacija i Kompromisno Resenje) introduced by Opricovic [30], ELECTRE (Elimination et choice translating reality) was introduced by Benayoun et al. [4], TOPSIS (Technique for Order Preference by similarity to an ideal solution) method was given by Hwang and Yoon [16] and Brans [7] suggested PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations). Each method has its own pros and cons. Thus, extended VIKOR method was suggested by Opricovic and Tzeng [29] and specified the limitations of PROMETHEE and ELECTRE. Compromise solution was attained by Opricovic and Tzeng [29] makes it more practical for different applications. Various authors ([32], [27], [1], [10]) employed VIKOR method but they used distance measure. The output of distance measure based methods vary with distance measures as in case of Joshi [21]. For selecting the best alternative some of the authors used accuracy function. But Ye [45] stated that accuracy function does not give enough information regarding alternative.

Distinct researchers Lin [25], Liu [15] and Boran [11] considered that weight determination is main exploration in fuzzy MCDM problem. Thus, we proposed a new entropy in IF environment and also suggested correlation coefficient based VIKOR approach. Although, various methods are available to solve MCDM problem as mentioned in literature still we adopt VIKOR method because it can used for contrary and incomparable criteria. It also provide the compromise solution which is closest to the ideal solution. An IF-VIKOR method was suggested by Xiao and Xuanzi [40] and used the distance measure to find the individual regret and group utility whereas we used correlation coefficient between distinct alternatives with positive and negative ideal solution to find best solutions. Thus, the main contribution of this paper:-

- The mathematics modeling of generalization of Shannon entropy has been introduced for probability distribution.
- Thereafter, we proposed an entropy measure for intuitionistic point of view and also examine their properties.

- We introduced a new term i.e. correlation coefficient and used this term instead of distance measure.
- We proposed extended IF- VIKOR method using the concept of correlation coefficient to evaluate the ranking sequence and also provided sensitive analysis to show the reliability.
- Further, comparison has been done with existing measures who used distance measure based VIKOR method.

The rest of the paper is structured as follows:- Section 2 covers the basic definitions related associated with fuzzy / intuitionistic fuzzy set. In section 3, we define correlation coefficient and proposed a new entropy measure for intuitionistic fuzzy set and also describes the literature associated with it. In the next section 4, we define algorithm for determining the criteria weights with the help of extended VIKOR method based on correlation coefficient. In Section 5, we presented the application of extended VIKOR method by taking numerical example and also provided sensitive analysis. The comparative analysis is also provided in this section. Finally, last section provided conclusion with future scope of improvement.

2. Preliminaries

Definition 2.1 (Zadeh [47]). Let us consider a non-empty set $Y = (y_1, y_2, \ldots, y_n)$, FS $\tilde{S}$ can be taken as:

$$\tilde{S} = \{\langle y_i, \mu_{\tilde{S}}(y_i)\rangle \mid y_i \in Y\}, \tag{2.1}$$

$\mu_{\tilde{S}} : Y \to [0, 1]$ represent membership function and $\mu_{\tilde{S}}(y_i) \in [0, 1]$ denote as membership degree of $y_i \in Y$ in $\tilde{S}$.

Atanassov [3] added another term called as "Hesitancy degree" and thus presents the "Intuitionistic Fuzzy Set (IFS)" which is further extension of fuzzy Set.

Definition 2.2 (Atanassov [3]). For a universal set $Y = (y_1, y_2, \ldots, y_n)$, IFS (Intuitionistic Fuzzy Set) $\tilde{S}$ can be expressed as:

$$\tilde{S} = \{\langle y_i, \mu_{\tilde{S}}(y_i), \nu_{\tilde{S}}(y_i)\rangle \mid y_i \in Y\}, \tag{2.2}$$

where $\mu_{\tilde{S}}(y_i)$ and $\nu_{\tilde{S}}(y_i)$ denotes membership and non-membership degrees of $y_i \in Y$ in $\tilde{S}$ that fulfill $0 \le \mu_{\tilde{S}}(y_i) + \nu_{\tilde{S}}(y_i) \le 1$. The number $\pi_{\tilde{S}}(y_i) = 1 - \mu_{\tilde{S}}(y_i) - \nu_{\tilde{S}}(y_i)$ represent the intuitionistic index or hesitancy degree. For $\pi_{\tilde{S}}(y_i) = 0$, then IFS will be converted in to FSs. Intuitionistic Fuzzy Number (IFN) can be represented as $\lambda = (\mu_\lambda, \nu_\lambda)$, where μ_λ and ν_λ lies in $[0, 1]$ with $\mu_\lambda + \nu_\lambda \le 1$. $\tilde{S}(\lambda) = \mu_\lambda - \nu_\lambda$ and $\tilde{H}(\lambda) = \mu_\lambda + \nu_\lambda$ represents "score value" and "accuracy degree" of λ, respectively.

Definition 2.3 (Xu [43] and Yager [42]). Let IFNs $\lambda_1 = (\mu_{\lambda_1}, \nu_{\lambda_1})$, $\lambda_2 = (\mu_{\lambda_2}, \nu_{\lambda_2})$ and $\lambda_3 = (\mu_{\lambda_3}, \nu_{\lambda_3})$, the different operations can be described as:

1. $\lambda_1 + \lambda_2 = (\mu_{\lambda_1} + \mu_{\lambda_2} - \mu_{\lambda_1}\mu_{\lambda_2}, \nu_{\lambda_1}\nu_{\lambda_2})$,
2. $\lambda_1 * \lambda_2 = (\mu_{\lambda_1}\mu_{\lambda_2}, \nu_{\lambda_1} + \nu_{\lambda_2} - \nu_{\lambda_1}\nu_{\lambda_2})$,
3. $\beta\lambda = (1-(1-\mu_\lambda)^\beta, (\nu_\lambda)^\beta)$; $\beta > 0$,
4. $\lambda^\beta = ((\mu_\lambda)^\beta, 1-(1-\nu_\lambda)^\beta)$; $\beta > 0$.

Definition 2.4 (Xia [39]). Let $\lambda_i = (\mu_{\lambda_i}, \nu_{\lambda_i})$; where $(i = 1, 2, \ldots, n)$ be a group of IFNs. Suppose $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ be the weight vector of λ_i $(i = 1, 2, \ldots, n)$ where $\lambda_i \in [0, 1]$ fulfilling the condition $\sum_{i=1}^n \lambda_i = 1$. Function SIFWA : $U^n \to U$ defined as

$$\begin{aligned}\mathrm{SIFWA}(\lambda_1, \lambda_2, \ldots, \lambda_n) =& \lambda_1\lambda_1 + \lambda_2\lambda_2 + \cdots + \lambda_n\lambda_n \\ =& \Big(\frac{\prod_{i=1}^n \mu_{\lambda_i}^{\lambda_i}}{\prod_{i=1}^n \mu_{\lambda_i}^{\lambda_i} + \prod_{i=1}^n (1-\mu_{\lambda_i})_i^{\lambda}}, \frac{\prod_{i=1}^n \nu_{\lambda_i}^{\lambda_i}}{\prod_{i=1}^n \nu_{\lambda_i}^{\lambda_i} + \prod_{i=1}^n (1-\nu_{\lambda_i})_i^{\lambda}}\Big).\end{aligned} \tag{2.3}$$

is known as Symmetric Intuitionistic Fuzzy Weighted Averaging (“ SIFWA”) operator.

Definition 2.5 (Atanassov [3]). **(Different operation (IFSs)** For any $\hat{T}, \hat{S} \in \mathrm{IFS}(Y)$ defined by:

$$\hat{T} =\{\langle y_i, \mu_{\hat{T}}(y_i), \nu_{\hat{T}}(y_i)\rangle \mid y_i \in Y\}, \tag{2.4}$$
$$\hat{S} =\{\langle y_i, \mu_{\hat{S}}(y_i), \nu_{\hat{S}}(y_i)\rangle \mid y_i \in Y\}; \tag{2.5}$$

different operations and their relationship can be described as:

1. $\hat{T} = \hat{S}$ iff $\hat{T} \subseteq \hat{S}$, that is $\mu_{\hat{T}}(y_i) \le \mu_{\hat{S}}(y_i)$ and $\nu_{\hat{T}}(y_i) \ge \nu_{\hat{S}}(y_i)$ for $\mu_{\hat{S}}(y_i) \le \nu_{\hat{S}}(y_i)$, or if $\mu_{\hat{T}}(y_i) \ge \nu_{\hat{S}}(y_i)$ and $\nu_{\hat{T}}(y_i) \le \nu_{\hat{S}}(y_i)$, for $\mu_{\hat{S}}(y_i) \ge \nu_{\hat{S}}(y_i)$ for any $y_i \in Y$.
2. $\hat{T} = \hat{S}$ iff $\hat{T} \subseteq \hat{S}$ and $\hat{S} \subseteq \hat{T}$;
3. $\hat{T}^c = \{\langle y_i, \nu_{\hat{T}}(y_i), \mu_{\hat{T}}(y_i)\rangle \mid y_i \in Y\}$;
4. $\hat{T} \cap \hat{S} = \{\langle \mu_{\hat{T}}(y_i) \wedge \mu_{\hat{S}}(y_i)$ and $\nu_{\hat{T}}(y_i) \vee \nu_{\hat{S}}(y_i)\rangle \mid y_i \in Y\}$;
5. $\hat{T} \cup \hat{S} = \{\langle \mu_{\hat{T}}(y_i) \vee \mu_{\hat{S}}(y_i)$ and $\nu_{\hat{T}}(y_i) \wedge \nu_{\hat{S}}(y_i)\rangle \mid y_i \in Y\}$.

3. A New Information Measure for IFSs

Let $\Gamma_n = \{\Omega = (\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n) : \overline{\omega}_i \ge 0,\ i = 1, 2, \ldots, n;\ \sum_{i=1}^n \omega_i = 1\}, n \ge 2$ be set of discrete probability distribution. For any probability distribution $\Omega = (\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n) \in \Gamma_n$, information measure was suggested by Shannon [33]

$$H_{\text{Shannon}}(\Omega) = -\sum_{i=1}^n (\overline{\omega}_i)\log(\overline{\omega}_i); \tag{3.1}$$

Generalization of Shannon [33] introduced by Renyi's [31] as:

$$H_{\text{Renyi}}(\Omega) = \begin{cases} \frac{1}{1-\beta}\left[\log\left(\sum_{i=1}^n \overline{\omega}_i^\beta\right)\right], & \beta > 0 (\ne 1); \\ -\sum_{i=1}^n (\overline{\omega}_i)\log(\overline{\omega}_i), & \beta = 1; \end{cases} \tag{3.2}$$

Remark. 1. If we put $\beta = 2$ in above equation (3.2) then it becomes Renyi Index.

$$\text{i.e.,} \quad H^{\beta=2}_{\text{Renyi}}(\Omega) = \log_D \Big(\sum_{i=1}^{n} \overline{\omega}_i^2\Big)^{-1}. \tag{3.3}$$

2. If $\beta \to \infty$, the Renyis entropy H_β approaches to H_∞ (Minimum Entropy).

$$H_\infty(\Omega) = -\log(\max(p_i)). \tag{3.4}$$

Although, various generalization of Shannon [33] suggested by different authors still scope of improvement is there. Therefore, we proposed a new information measure ${}_\beta H_{\text{new}}(\Omega)$: $\Gamma_n \to \mathfrak{R}^+$ (set of positive real numbers); $n \geq 2$ as follows:

$${}_\beta H_{\text{new}}(\Omega) = \begin{cases} \frac{1}{\beta^{-1}-\beta} \log\Big(\frac{\sum_{i=1}^{n} \overline{\omega}_i^{\beta}}{\sum_{i=1}^{n} \overline{\omega}_i^{\beta-1}}\Big), & \beta > 0 (\neq 1); \\ -\sum_{i=1}^{n} (\overline{\omega}_i) \log(\overline{\omega}_i), & \beta = 1. \end{cases} \tag{3.5}$$

3. ${}_\beta H_{\text{new}}(\Omega) = {}_{\beta^{-1}} H_{\text{new}}(\Omega)$, that is, (3.5) is symmetric w.r.t (β, β^{-1}).
4. Relationship between proposed measure (3.5) and Renyi's entropy :

$$\begin{aligned} {}_\beta H_{\text{new}}(\Omega) &= \frac{1}{\beta^{-1}-\beta}\Big[\log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta}\Big) - \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta^{-1}}\Big)\Big] \\ &= \frac{\beta}{1-\beta^2}\Big[\frac{1-\beta}{1-\beta} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta}\Big) - \frac{1-\beta^{-1}}{1-\beta^{-1}} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta^{-1}}\Big)\Big] \\ &= \frac{\beta}{(1+\beta)(1-\beta)}\Big[\frac{1-\beta}{1-\beta} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta}\Big) + \frac{\beta^{-1}-1}{1-\beta^{-1}} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta^{-1}}\Big)\Big] \\ &= \frac{\beta}{(1+\beta)}\Big[\frac{1}{1-\beta} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta}\Big) + \frac{1}{\beta-1} \log\Big(\sum_{i=1}^{n} \overline{\omega}_i^{\beta^{-1}}\Big)\Big]. \end{aligned}$$

Therefore, we get

$${}_\beta H_{\text{new}}(\Omega) = \frac{\beta}{(1+\beta)}\Big[H^{\beta}_{\text{Renyi}}(\Omega) + H^{\beta^{-1}}_{\text{Renyis}}(\Omega)\Big]. \tag{3.6}$$

This implies the proposed measure ${}_\beta H_{\text{new}}(s)$ is equal to the constant $(\frac{\beta}{1+\beta})$ times sum of Renyi's [31] entropy. So, there is a close relationship between (3.5) and (3.2) which are already existed in literature.

Generalized Measure axioms proposed in (3.5)

Theorem 3.1. *For any* $\Omega \in \Gamma_n$, *and* ${}_\beta H_{\text{new}}(\Omega)$ *satisfies the following properties :*

a. ${}_\beta H_{\text{new}}(\Omega) \geq 0$ *for all* $\beta > 0 (\neq 1)$. [Non-negativity]
b. ${}_\beta H_{\text{new}}(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)$ *is a symmetric function of* $(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)$.

c. ${}_\beta H_{\text{new}}(0,1) = 0 =_\beta H_{\text{new}}(1,0)$. [Decisivity]

d. *For any* $\Omega = (\varpi_1, \varpi_2, \ldots, \varpi_n) \in \Gamma_n$, *we have* ${}_\beta H_{\text{new}}(\Omega) =_\beta H_{\text{new}}(\varpi_1, \varpi_2, \ldots, \varpi_n, 0)$. [Expandability]

e. *For* $\Omega = (\varpi_1, \varpi_2, \ldots, \varpi_n) \in \Gamma_n$ *and* $\Xi = (\xi_1, \xi_2, \ldots, \xi_m) \in \Gamma_m$, *let us define their independent combination as* $\Omega * \Xi = (\varpi_i \xi_j)_{i=1,\ldots,n; j=1,\ldots,m}$. *Then,* ${}_\beta H_{\text{new}}(\Omega * \Xi) = {}_\beta H_{\text{new}}(\Omega) + {}_\beta H_{\text{new}}(\Xi)$. [Shannon additivity/ Extensivity]

f. ${}_\beta H_{\text{new}}(\frac{1}{2}, \frac{1}{2}) = 1$. [Normalize]

g. ${}_\beta H_{\text{new}}(\varpi_1, \varpi_2, \ldots, \varpi_n) \leq {}_\beta H_{\text{new}}(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}) = \log(n)$.

h. ${}_\beta H_{\text{new}}(\varpi_1, \varpi_2, \ldots, \varpi_n)$ *is continuous in* ϖ_i*'s* $\forall\ i = 1, 2, \ldots, n$ *and* $\beta > 0$.

3.1. FSs and IFSs entropy

Definition 3.1 (Zadeh [46]). A function $\tilde{\Phi} : FS(Y) \to [0, \infty)$ can be defined as fuzzy entropy when it fulfills the following axioms:

1. $\tilde{S}$ is crisp set iff $\tilde{\phi}(\tilde{S}) = 0$, for all $\tilde{S} \in FS(Y)$.
2. If $\mu_{\tilde{S}} = 0.5$ iff $\tilde{\phi}(\tilde{S})$ is maximum $\forall\ \tilde{S} \in FS(Y)$.
3. For any $\tilde{T}, \tilde{S} \in FS(Y)$, $\tilde{\phi}(\tilde{T}) \leq \tilde{\phi}(\tilde{S})$ if $\tilde{T}$ is crisper than $\tilde{S}$, that is, $\mu_{\tilde{T}} \leq \mu_{\tilde{S}}$ if $\mu_{\tilde{S}} \leq 0.5$ and $\mu_{\tilde{T}} \geq \mu_{\tilde{S}}$ if $\mu_{\tilde{S}} \geq 0.5$.
4. $\tilde{\phi}(\tilde{S}) = \tilde{\phi}(\tilde{S})^c$; $(\tilde{S})^c$ indicate complement of $\tilde{S} \in FS(Y)$.

Further, Hung and Yang [17] suggested a new entropy by expanding the concept given by Luca and Termini [9].

Definition 3.2 (Atanassov [3]). A real function $\overline{\Phi}$: IFS$(Y) \to [0, \infty)$ is known as an entropy on IFS(Y) if the following properties are satisfies:

1. $\hat{S}$ is crisp set iff $\overline{\phi}(\hat{S}) = 0$.
2. The value of $\overline{\phi}(\hat{S})$ is maximum at $\mu_{\hat{S}} = \nu_{\hat{S}} = \pi_{\hat{S}} = \frac{1}{3}$.
3. $\overline{\phi}(\hat{T}) \leq \overline{\phi}(\hat{S})$ iff $\hat{T}$ is crisper than $\hat{S}$, i.e., $\mu_{\hat{T}} \geq \mu_{\hat{S}}$, $\nu_{\hat{T}} \geq \nu_{\hat{S}}$ if $\min(\mu_{\hat{T}}, \nu_{\hat{S}}) \geq \frac{1}{3}$, and $\mu_{\hat{T}} \leq \mu_{\hat{S}}$, $\nu_{\hat{T}} \leq \nu_{\hat{S}}$ if $\max(\mu_{\hat{S}}, \nu_{\hat{S}}) \leq \frac{1}{3}$.
4. $\overline{\phi}(\hat{S}) = \overline{\phi}(\hat{S})^c$ where $(\hat{S})^c$ indicates its complement.

Definition 3.3 (Szmidt [34]). An entropy on IFS(Y) is a real-valued function A: IFS$(Y) \to [0, 1]$, that satisfy the following axiom:

$\aleph_1$. $A(\hat{T}) = 1$ iff $\mu_{\hat{T}}(y_i) = \nu_{\hat{T}}(y_i)$, for all $y_i \in Y$.

$\aleph_2$. $A(\hat{T}) = 0$ iff $\hat{T}$ is crisp set; i.e., $\mu_{\hat{T}}(y_i) = 0$, $\nu_{\hat{T}}(y_i) = 1$ or $\mu_{\hat{T}}(y_i) = 1$, $\nu_{\hat{T}}(y_i) = 0$ for all $y_i \in Y$.

$\aleph_3$. $A(\hat{T}) \leq A(\hat{S})$ iff $\hat{T} \subseteq \hat{S}$.

$\aleph_4$. $A(\hat{T}) = A(\hat{T})^c$.

Definition 3.4 (Co-relation Coefficients [13])**.** Suppose $\hat{S}_1 = \{\langle y_i, \mu_{\hat{S}_1}(y_i), \nu_{\hat{S}_1}(y_i)\rangle \mid y_i \in Y\}$ and $\hat{S}_2 = \{\langle y_i, \mu_{\hat{S}_2}(y_i), \nu_{\hat{S}_2}(y_i)\rangle \mid y_i \in Y\}$ are two IFSs. Gerstenkorn Manko define correlation coefficient $\mathfrak{J}(\hat{S}_1, \hat{S}_2)$ between $\hat{S}_1$ and $\hat{S}_2$ as follows:

$$\mathfrak{J}(\hat{S}_1, \hat{S}_2) = \frac{\mathsf{F}(\hat{S}_1, \hat{S}_2)}{\sqrt{\phi(\hat{S}_1)\phi(\hat{S}_2)}}; \tag{3.7}$$

where

$$\mathsf{F}(\hat{S}_1, \hat{S}_2) = \sum_{i=1}^{n} \Big(\mu_{\hat{S}_1}(y_i)\mu_{\hat{S}_2}(y_i) + \nu_{\hat{S}_1}(y_i)\nu_{\hat{S}_2}(y_i)\Big); \tag{3.8}$$

represents the co-relation between two IFSs $\hat{S}_1$ and $\hat{S}_2$, where

$$\psi(\hat{S}_1) = \sum_{i=1}^{n} \Big((\mu_{\hat{S}_1}(y_i))^2 + (\nu_{\hat{S}_1}(y_i))^2\Big), \tag{3.9}$$

$$\psi(\hat{S}_2) = \sum_{i=1}^{n} \Big((\mu_{\hat{S}_2}(y_i))^2 + (\nu_{\hat{S}_2}(y_i))^2\Big); \tag{3.10}$$

The co-relation coefficient $\mathsf{F}(\hat{S}_1, \hat{S}_2)$ satisfies the following properties:

1. $0 \le \mathsf{F}(\hat{S}_1, \hat{S}_2) \le 1$.
2. $\mathsf{F}(\hat{S}_1, \hat{S}_2) = \mathsf{F}(\hat{S}_2, \hat{S}_1)$.
3. $\mathsf{F}(\hat{S}_1, \hat{S}_2) = 1$ if $\hat{S}_1 = \hat{S}_2$.

With the above ideas, we will propose a new entropy for Intuitionistic Fuzzy Set. Luca and Termini [9] suggested entropy relative to Shannon entropy as:

$$H_{LT}(\tilde{S}) = \frac{1}{n}\sum_{i=1}^{n} \Big[\mu_{\tilde{S}}(y_i)\log(\mu_{\tilde{S}}(y_i)) + (1 - \mu_{\tilde{S}}(y_i)\log(1 - \mu_{\tilde{S}}(y_i)))\Big]; \tag{3.11}$$

where $\tilde{S} \in FS(Y)$ and $y_i \in Y$.

Renyi's idea was further extended by Bhandari and Pal [5] and suggested his entropy as:

$$H_{BP}(\tilde{S}) = \frac{1}{n(1-\beta)}\sum_{i=1}^{n} \log\Big[(\mu_{\tilde{S}}(y_i))^{\beta} + (1 - \mu_{\tilde{S}}(y_i))^{\beta}\Big]. \tag{3.12}$$

With the further extension of Verma and Sharma [35], now, we proposed a new information measure for IFSs.

Definition 3.5. For any $\hat{T} \in \mathrm{IFS}(Y)$, we define :

$$A_{\beta}(\tilde{T}) = \frac{1}{n(\beta^{-1} - \beta)}$$

$$\times\sum_{i=1}^{n}\log\Big[\frac{[(\mu_{\hat{T}}(y_i))^{\beta}+(\nu_{\hat{T}}(y_i))^{\beta}]\times(\mu_{\hat{T}}(y_i)+\nu_{\hat{T}}(y_i))^{(1-\beta)}+2^{1-\beta}\pi_{\hat{T}}(y_i)}{[(\mu_{\hat{T}}(y_i))^{\beta-1}+(\nu_{\hat{T}}(y_i))^{\beta-1}]\times(\mu_{\hat{T}}(y_i)+\nu_{\hat{T}}(y_i))^{(1-\beta^{-1})}+2^{1-\beta^{-1}}\pi_{\hat{T}}(y_i)}\Big];$$

$$\beta > 0(\neq 1). \tag{3.13}$$

Then (3.13) is a proposed information measure for intuitionistic fuzzy environment.

Particular cases

- For $\beta = 1$, then (3.13) converts in to Vlachos and Sergiadis [36] entropy.
- For $\beta = 1$ and $\pi_{\hat{T}}(y_i) = 0$, then (3.13) will convert in to Luca and Termini [9] entropy.
- For $\pi_{\hat{T}}(y_i) = 0$, then (3.13) becomes fuzzy information measures relative to (3.5).
- $A_\beta(\hat{T}) = A_{\beta^{-1}}(\hat{T})$, i.e., (3.13) is symmetric for intuitionistic case.

Lemma. *The measure $A_\beta(\hat{T})$ in* (3.13) *is a valid IF-entropy having order β and satisfying the axioms $\aleph_1 - \aleph_4$.*

4. Extended VIKOR Method Based on Correlation Coefficient

Opricovic [30] suggested to determine the best alternative of compromise solution close to the ideal solution. Thus to find the solution it is important to determine criteria weight. Criteria weights can be categorized as subjective and objective criteria weights. This section further describes the algorithm to calculate the criteria weight using proposed entropy.

4.1. Algorithm to determine criteria weights

As we know that criteria weights plays an important part in MCDM problems. To determine the criteria weights algorithm as follows:

1. Let us construct MCDM problem in the form of $m \times n$ matrix where m denotes the alternative $(\varphi_i)_{1\leq i\leq m}$ and n denotes the criteria $(\varrho_j)_{1\leq j\leq n}$
   Let us consider IF- decision matrix as:

$$D = [d_{ij}]_{m\times n} = \begin{array}{c} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_m \end{array}\begin{array}{c} \begin{array}{cccc} \varrho_1 & \varrho_2 & \cdots & \varrho_n \end{array} \\ \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix} \end{array} \tag{4.1}$$

   where $d_{ij} = (\overline{\mu}_{ij}, \overline{\nu}_{ij}); i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.
2. In this step, we will obtain F_j; $j = 1, 2, \ldots, n$ by using (3.13).

Now, we will discuss two conditions to evaluate the criteria weights.

(a) Criteria Weights (Partially Known)

Although, many conditions are there to determine criteria weights. We need experts for evaluating criteria weight. A single expert can not expertise all the fields and hiring experts for different fields will be very costly. It is easy for them to give their opinion in a form which is different than perfect number such as linguistic variable. Some of the information about criteria weight present with us and the whole information can be gathered by a set designated as W^T. After that idea of minimum entropy [38] was incorporated to evaluate the criteria weight from W^T.

Entropy value of alternatives φ_i which covers the criteria ϱ_j can specified as:

$$F_j = \sum_{i=1}^{n} F_{\beta}^{\beta^{-1}}(d_{ij}). \tag{4.2}$$

Where

$$F_{\beta}^{\beta^{-1}}(d_{ij}) = \frac{1}{n(\beta^{-1}-\beta)} \times \sum_{i=1}^{m} \log\left[\frac{[(\mu_T(y_i))^{\beta}+(\nu_T(y_i))^{\beta}]\times(\mu_T(y_i)+\nu_T(y_i))^{(1-\beta)}+2^{1-\beta}\pi_T(y_i)}{[(\mu_T(y_i))^{\beta-1}+(\nu_T(y_i))^{\beta-1}]\times(\mu_T(y_i)+\nu_T(y_i))^{(1-\beta^{-1})}+2^{1-\beta^{-1}}\pi_T(y_i)}\right]. \tag{4.3}$$

Now, we will construct linear programming model to determine the optimal criteria weights which is as follows:

$$\begin{aligned} \min\ F &= \sum_{i=1}^{m} F_j(\varphi_i) = \sum_{i=1}^{m}\left[\sum_{j=1}^{n}\zeta_j F_{\beta}^{\beta^{-1}}(d_{ij})\right] \\ &= \frac{1}{n(\beta^{-1}-\beta)}\sum_{i=1}^{m}\sum_{j=1}^{n}\zeta_j \\ &\quad \times \log\left[\frac{[(\mu_T(y_i))^{\beta}+(\nu_T(y_i))^{\beta}]\times(\mu_T(y_i)+\nu_T(y_i))^{(1-\beta)}+2^{1-\beta}\pi_T(y_i)}{[(\mu_T(y_i))^{\beta-1}+(\nu_T(y_i))^{\beta-1}]\times(\mu_T(y_i)+\nu_T(y_i))^{(1-\beta^{-1})}+2^{1-\beta^{-1}}\pi_T(y_i)}\right], \end{aligned} \tag{4.4}$$

which fulfilling $\sum_{j=1}^{n}\zeta_j = 1$, $\zeta_j \in W^T$. After solving (4.4), the criteria weights vector can be described as arg min $F = (\zeta_1, \zeta_2, \ldots, \zeta_n)'$.

(b) Criteria Weights (For unknown)

For calculating the unknown criteria weights we use the equation (4.5) suggested by [8].

$$\zeta_j = \frac{1-F_j}{n-\sum_{j=1}^{n}F_j}; \qquad j = 1, 2, \ldots, n; \tag{4.5}$$

where

$$F_{\beta}^{\beta^{-1}}(d_{ij}) = \frac{1}{n(\beta^{-1}-\beta)}$$

$$\times \sum_{i=1}^{m} \log \Big[\frac{[(\mu_T(y_i))^{\beta} + (\nu_T(y_i))^{\beta}] \times (\mu_T(y_i) + \nu_T(y_i))^{(1-\beta)} + 2^{1-\beta}\pi_T(y_i)}{[(\mu_T(y_i))^{\beta-1} + (\nu_T(y_i))^{\beta-1}] \times (\mu_T(y_i) + \nu_T(y_i))^{(1-\beta^{-1})} + 2^{1-\beta^{-1}}\pi_T(y_i)} \Big]. \tag{4.6}$$

4.2. VIKOR method

In this section, we introduce step wise procedure for extended VIKOR method using correlation coefficient. Let us consider EY_j are the different decision makers where $(j = 1, 2, \ldots, n)$ decides the best alternative from a set of alternatives φ_i $(i = 1, 2, \ldots, m)$. Each decision makers have different weights ω_j $(j = 1, 2, \ldots, n)$ which satisfying $\sum_{j=1}^{n} \omega_j = 1$. The different steps of extended VIKOR method are summarized as:

(i) IF Decision Matrix construction

Let $d_{ij}^{p} = (\mu_{ij}^{p}, \nu_{ij}^{p})$ is intuitionistic fuzzy number (IFN) specified by different decision makers. In this step, the different entries of decision matrix can be obtained with the help of following equation:

$$\begin{aligned} d_{ij}^{p} &= \sum_{p=1}^{k} \omega_p d_{ij}^{p} \\ &= \Big(\frac{\prod_{p=1}^{k} (\mu_{ij}^{p})^{\omega_p}}{\prod_{p=1}^{k} (\mu_{ij}^{p})^{\omega_p} + \prod_{p=1}^{k} (1 - \mu_{ij}^{p})^{\omega_p}}, \frac{\prod_{p=1}^{k} (\nu_{ij}^{p})^{\omega_p}}{\prod_{p=1}^{k} (\nu_{ij}^{p})^{\omega_p} + \prod_{p=1}^{k} (1 - \nu_{ij}^{p})^{\omega_p}} \Big); \end{aligned} \tag{4.7}$$

where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

(ii) Normalization of Decision Matrix

In this step, we normalize the output obtained from equation (4.7) by means of equation (4.8) which was suggested by Xu and Hu [43].

$$\tilde{t}_{ij} = \sum_{j=1}^{n} \frac{\Im(\Upsilon_j^{++}, d_{ij})}{\Im(\Upsilon_j^{++}, \Upsilon_j^{--})}; \tag{4.8}$$

(iii) Calculation of Subjective weights

Let $\zeta_j^{p} = (\mu_j^{p}, \nu_j^{p})$ are the different weights specified by different decision makers (EY^{p}). Thus, we will calculate subjective weight (ζ_j^{p}) with the help of operator ("SIFWA").

$$\begin{aligned} \zeta_j &= \text{SIFWA}(\zeta_j^{1}, \zeta_j^{2}, \ldots, \zeta_j^{k}) = \sum_{p=1}^{k} \omega_p \zeta_j^{p} \\ &= \Big(\frac{\prod_{p=1}^{k} (\mu_j^{p})^{\omega_p}}{\prod_{p=1}^{k} (\mu_j^{p})^{\omega_p} + \prod_{p=1}^{k} (1 - \mu_j^{p})^{\omega_p}}, \frac{\prod_{p=1}^{k} (\nu_j^{p})^{\omega_p}}{\prod_{p=1}^{k} (\nu_j^{p})^{\omega_p} + \prod_{p=1}^{k} (1 - \nu_j^{p})^{\omega_p}} \Big). \end{aligned} \tag{4.9}$$

Where $\zeta_j = (\mu_j, \nu_j)$, $j = 1, 2, \ldots, n$.

(iv) Normalization of Subjective weights

In the step, we normalize the subjective weight suggested by (Li [23], Boran [6]) satisfying $\sum_{j=1}^{n}\zeta_j^z = 1$.

$$\zeta_j^z = \frac{\mu_j + \tau_j\left(\frac{\mu_j}{\mu_j+\nu_j}\right)}{\sum_{j=1}^{n}\left[\mu_j + \tau_j\left(\frac{\mu_j}{\mu_j+\nu_j}\right)\right]}; \tag{4.10}$$

where $\tau_j = 1 - \mu_j - \nu_j$.

(v) Objective weights calculation

Objective weights ζ_j^b can also calculated using the above described equation (4.7) and (4.8).

(vi) Determine best Solution and Cost Criteria

In this step, we determine the IF best $(\Upsilon_j^{++} = (\mu_j^{++}, \nu_j^{++}))$ and worst solution $(\Upsilon_j^{--} = (\mu_j^{--}, \nu_j^{--}))$ using the following equations.

$$\Upsilon_j^{++} = \begin{cases} \max_i d_{ij}, & \text{for benefit criteria} \\ \min_i d_{ij}, & \text{for cost criteria;} \end{cases} \tag{4.11}$$

and

$$\Upsilon_j^{--} = \begin{cases} \min_i d_{ij}, & \text{for benefit criteria} \\ \max_i d_{ij}, & \text{for cost criteria;} \end{cases} \tag{4.12}$$

(vii) Calculation of T_i and R_i

Let us find the values of T_i, R_i, where $i = 1, 2, \ldots, m$ using equation (4.13) and (4.14).

$$T_i = \Theta\sum_{j=1}^{n}\zeta_j^z\tilde{t}_{ij} + (1-\Theta)\zeta_j^b\tilde{t}_{ij} = \sum_{j=1}^{n}\widehat{w_j}\tilde{t}_{ij}, \tag{4.13}$$

and

$$R_i = \max(\widehat{w_j}\tilde{t}_{ij}), \tag{4.14}$$

where T_i = group utility, R_i = individual regret, $\widehat{w_j} = \sum_{j=1}^{n}(\Theta\zeta_j^z + (1-\Theta)\zeta_j^b)$ is the merger of objective and subjective weights and Θ presents the relation between objective and subjective weights i.e., $\Theta \in [0,1]$ and $\Theta = 0.5$.

(viii) Determine the VIKOR index (Q_i)

$$Q_i = \Psi\frac{T_i - T^{--}}{T^{++} - T^{--}} + (1-\Psi)\frac{R_i - R^{--}}{R^{++} - R^{--}}, \tag{4.15}$$

where Ψ and $(1-\Psi)$ are represents the weights for T_i and R_i. We consider the value of Ψ as 0.5. In the above equation, we consider T^{++} as maximum value of T_i and T^{--}

as minimum value of T_i. In the similar fashion R^{++} as maximum value of R_i and R^{--} as minimum value of R_i. After computing the values using equation (4.13), (4.14) and (4.15) arrange these values in ascending order and also rank the alternatives. Following are the condition that should be satisfied by alternatives.

$\overline{\boldsymbol{C}}$**1 (Acceptable Advantage)** If $Q(\Upsilon^2 - \Upsilon^1) \geq \frac{1}{n-1}$ where Υ^1 and Υ^2 are first and second ranked of alternative respectively.

$\overline{\boldsymbol{C}}$**2 (Acceptable Stability)** For R_i and T_i, Υ^1 should also ranked first.

If both of the above conditions satisfied simultaneously for a particular alternative then that alternative will be considered as most desirable otherwise we will go for compromise solution which is as follows:

- (Υ^1, Υ^2) will be set of compromise solution if $\overline{\boldsymbol{C}}2$ is not satisfied.
- If $\overline{C}1$ condition does not meet then $(\Upsilon^1, \Upsilon^2, \ldots, \Upsilon^M)$ constitute the compromise solution where Υ^M is defined by

$$Q(\Upsilon^M) - Q(\Upsilon^1) < \frac{1}{n-1}. \tag{4.16}$$

Where M = maximum of ranks of the alternatives and n = total number of criteria.

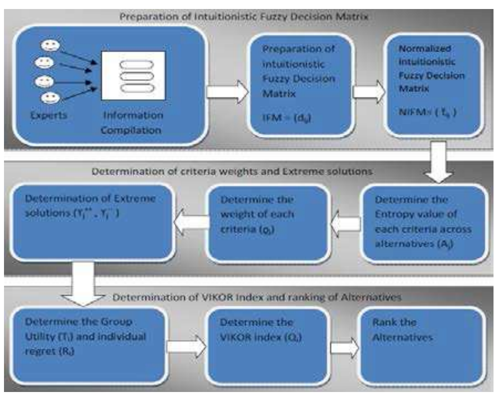

Figure 1: Flowchart of decision making method.

5. Application in MCDM Problem

In this section, we will solve decision making problem using the concept of extended VIKOR method. We have considered two approaches to solve the numerical example.

5.1. Approach 1: Criteria Weights (Known partially)

Consider a Textile industry wants to buy some parts of generator. There are five different suppliers or alternatives φ_i $(i = 1, 2, 3, 4, 5)$. For decision making industry takes opinion from five experts FY_i $(i = 1, 2, 3, 4, 5)$. They set five criteria such as Functionality (ϱ_1), Reliability (ϱ_2), Customer Satisfaction (ϱ_3), Quality (ϱ_4), Cost (ϱ_5)

Table 1: Rating of Alternative.

Linguistic Variables	IFNs
Very poor (VP)	(.15, .85)
Poor (P)	(.25, .75)
Moderately poor (MP)	(.30, .65)
Fair (F)	(.45, .55)
Moderately Good (MG)	(.55, .45)
Good (G)	(.75, .20)
Very Good (VG)	(.85, .15)

Table 2: Rating of criteria weights.

Linguistic Variables	IFNs
Very low (VL)	(.75, .10)
low (L)	(.25, .50)
Medium Low (ML)	(.35, .60)
Medium (M)	(.45, .65)
High (H)	(.55, .65)
Very high(VH)	(.70, .25)

Table 3: Output of Decision-maker's.

Criteria	Decision makers	Alternatives				
		φ_1	φ_2	φ_3	φ_4	φ_5
ϱ_1	EY_1	F	VG	MG	MP	G
	EY_2	MG	VG	G	G	G
	EY_3	MG	G	MG	F	VG
	EY_4	G	G	MG	F	VG
	EY_5	G	VG	MP	G	F
ϱ_2	EY_1	MG	G	MG	VG	MG
	EY_2	F	F	MG	G	VG
	EY_3	F	MG	VG	G	MG
	EY_4	G	VG	MG	G	VG
	EY_5	MG	G	VG	G	MG
ϱ_3	EY_1	F	F	MG	G	VG
	EY_2	G	MG	MG	VG	MG
	EY_3	G	MG	MG	VG	MG
	EY_4	G	MG	MG	VG	MG
	EY_5	G	VG	MP	G	G
ϱ_4	EY_1	G	G	G	G	G
	EY_2	F	MG	VG	VG	G
	EY_3	G	MG	MG	VG	F
	EY_4	F	MG	VG	VG	G
	EY_5	G	G	G	G	G
ϱ_5	EY_1	MG	VG	G	MG	G
	EY_2	G	F	F	VG	G
	EY_3	MG	VG	G	MG	G
	EY_4	G	F	F	VG	G
	EY_5	MG	VG	G	MG	G

to select the alternative. Tables 1 and Table 2 describes the rating of alternative and criteria weight respectively in the term of linguistic variable using intuitionistic fuzzy number (IFN). Decision maker's output that shows the relation between criteria and alternatives described in Table 3.

(i). Firstly, we will obtain IF- decision matrix using equation (4.7) and output obtained are as described in Table 4.

Table 4: IF-Decision Matrix.

	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
φ_1	(0.2855,0.0261)	(0.5686,0.5002)	(0.1307,0.1023)	(0.1991,0.0520)	(0.3979,0.0154)
φ_2	(0.1922,0.0684)	(0.3560,0.0174)	(0.3686,0.0184)	(0.4867,0.0078)	(0.3406,0.0275)
φ_3	(0.3916,0.0115)	(0.2623,0.0384)	(0.1080,0.1405)	(0.5715,0.0050)	(0.3026,0.0349)
φ_4	(0.2726,0.0300)	(0.2764,0.0370)	(0.4683,0.0098)	(0.5715,0.0050)	(0.3596,0.0158)
φ_5	(0.2742,0.0385)	(0.3752,0.0178)	(0.2726,0.0300)	(0.3406,0.0275)	(0.4550,0.0084)
A_j	0.7915	0.7967	0.8036	0.6078	0.6765

(ii). In this step, we calculate the values for subjective and normalized criteria weight using (4.9) and (4.10) as described in Table 5

Table 5: Subjective Criteria Weights and its Normalized Value.

Criteria weight	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
Subjective	(0.2789,0.0300)	(0.3625,0.0519)	(0.2441,0.0388)	(0.4241,0.0124)	(0.3697,0.0182)
Normalized	0.1978	0.1916	0.1890	0.2128	0.2088

(iv). Next, we have to calculate the objective weights. Thus, we define information set with its weights as:

$$\hat{M} = \{0.15 \leq \zeta_1 \leq 0.30, 0.10 \leq \zeta_2 \leq 0.25, 0.25 \leq \zeta_3 \leq 0.35, 0.30 \leq \zeta_4 \leq 0.45, \\ 0.20 \leq \zeta_5 \leq 0.40\}. \tag{5.1}$$

Firstly, construct the objective weights using programming model which is as below:

$$\text{Min } F = 0.7915\zeta_1 + 0.7967\zeta_2 + 0.8036\zeta_3 + 0.6078\zeta_4 + 0.6765\zeta_5;$$

$$= \text{subjectto} \begin{cases} 0.15 \leq \zeta_1 \leq 0.30; \\ 0.10 \leq \zeta_2 \leq 0.25; \\ 0.25 \leq \zeta_3 \leq 0.35; \\ 0.30 \leq \zeta_4 \leq 0.45; \\ 0.20 \leq \zeta_5 \leq 0.40; \\ \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5 = 1. \end{cases} \tag{5.2}$$

After solving equation (5.2), we obtain the objective criteria weight as: $\zeta = (0.15, 0.10, 0.25, 0.30, 0.20)$

(v). In this step, we normalize the decision matrix using equation (4.8). Table 6 describes the normalized value of decision matrix.

Table 6: Normalized IF-Decision Matrix ($\tilde{t}_{ij}$).

Alternatives	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
φ_1	0.8091	1.0016	1	0.8948	0.7770
φ_2	1	1.0549	1.0548	1.0555	1.0533
φ_3	1.6219	1.6072	1	1.6223	1.6132
φ_4	1	0.9973	1.0050	1.00512	1.0045
φ_5	0.9968	1.0038	1	1.0023	1.0042

(vi). We calculate the values of T_i, R_i, Q_i for all alternatives using equation (4.13), (4.14) and (4.15) as shown in Table 7

Table 7: Calculation of T, R and Q.

Values	φ_1	φ_2	φ_3	φ_4	φ_5
T	0.8945	1.0451	1.4816	1.0029	1.0014
R	0.2294	0.2706	0.4159	0.2577	0.2570
Q	0	0.2388	1	0.1682	0.1650

(vii). Finally, we sequence the alternatives depending upon the values of Table 7 as described in Table 8

Table 8: Alternatives sequence.

By T	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$
By R	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$
By Q	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$

After the observation of Table 8 we noticed that φ_1 and φ_5 are ranked as first and second for Q. Thus, according to condition $\overline{\boldsymbol{C}}1$, $Q(\varphi_5) - Q(\varphi_1) = 0.1650 - 0 = 0.1650 < \frac{1}{5-1} = 0.25$ so, it is not satisfied . But the condition $\overline{\boldsymbol{C}}2$ is satisfied as φ_1 has also ranked first for the case of T and R. Hence, we have find φ_1 is the best alternative.

5.2. Sensitive interpretation

Now, we will discuss the sensitive analysis to depict the proposed information behavior. It should be noted that by changing the values of weight Ψ fuzzy information should not change because it affects the reliability. Also, compromise solution should not change on different values of weight Ψ. Thus, we have calculated the ranking sequences of Q on different values of weight Ψ as shown in Table 9. After observing Table 9, we conclude that same ranking sequences are obtain on different values of Ψ. Hence, proposed entropy is reliable as ranking sequence of alternative remain same even on the different values of Ψ. The graphical output of Q on different values of Ψ can also demonstrated with the help of Figure 2.

Table 9: Values of T_i, R_i and Q_i obtained on changing weight (Ψ).

	Ψ	φ_1	φ_2	φ_3	φ_4	φ_5	Ranking	Compromise Solution
T		0.8945	1.0451	1.4816	1.0029	1.0014	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
R		0.2294	0.2706	0.4159	0.2577	0.2570	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
Q	0	0.0000	0.2209	1.0000	0.1517	0.1480	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.1	0.0000	0.2245	1.0000	0.1550	0.1514	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.2	0.0000	0.2280	1.0000	0.1583	0.1548	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.3	0.0000	0.2316	1.0000	0.1616	0.1582	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.4	0.0000	0.2352	1.0000	0.1649	0.1616	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.5	0.0000	0.2388	1.0000	0.1682	0.1650	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.6	0.0000	0.2423	1.0000	0.1715	0.1684	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.7	0.0000	0.2458	1.0000	0.1748	0.1719	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.8	0.0000	0.2494	1.0000	0.1781	0.1753	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	0.9	0.0000	0.2530	1.0000	0.1813	0.1787	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1
	1	0.0000	0.2565	1.0000	0.1846	0.1821	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$	φ_1

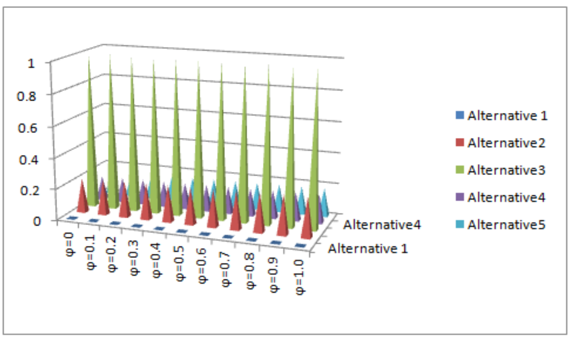

Figure 2: Sensitivity Outcomes.

5.3. Approach 2: For unknown criteria weights

Now, we will solve example mention in (5.1) for unknown criteria weight. The computational steps are as follows:

1. Calculate the values of criteria weights Using (4.5), which are stated as:

$$\zeta_1 = 0.1575, \quad \zeta_2 = 0.1536, \quad \zeta_3 = 0.1483, \quad \zeta_4 = 0.2962 \text{ and } \zeta_5 = 0.2444. \tag{5.3}$$

2. Using (4.11) and (4.12), we calculate the positive (Υ_j^{++}) and negative (Υ_j^{--}) ideal solution as described in Table 10.
3. We calculate the values of T_i, R_i and Q_i with the help of equations (4.13), (4.14) and (4.15) which are as mentioned in Table 11.
4. After observing Table 11, we can find the sequence of alternatives which are mentioned in Table 12.

After the observation of Table 12, we perceive that φ_1 and φ_5 are ranked as first and second in the sequence. Thus, $Q(\varphi_1) - Q(\varphi_5) = 0.5214 - 0 = 0.5214 > 1/(5-1) = 0.25$

Table 10: Positive (Υ_j^{++}) and Negative (Υ_j^{--}) ideal solution.

	ϱ_1	ϱ_2	ϱ_3	ϱ_4	ϱ_5
Υ_j^{++}	(0.3916,0.0115)	(0.5686,0.5002)	(0.4683,0.0098)	(0.5715,0.0050)	(0.4550,0.0084)
Υ_j^{--}	(0.1922,0.0684)	(0.2623,0.0384)	(0.1080,0.1405)	(0.1991,0.0520)	(0.3026,0.0349)

Table 11: Calculation of T, R and Q.

	φ_1	φ_2	φ_3	φ_4	φ_5
T	1.0121	1.1989	1.2154	1.1964	1.1452
R	0.4125	0.4487	0.4564	0.4401	0.4385
Q	0	0.5498	1	0.5388	0.5214

Table 12: Alternatives sequence.

By T	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$
By R	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$
By Q	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$

which states that $\overline{\boldsymbol{C}}1$ is satisfied. Similarly, for T and R, φ_1 has also ranked as first. Thus, $\overline{\boldsymbol{C}}2$ is also satisfied. So, the best alternative is φ_1. Hence, φ_1 is the most preferred alternative to supply the parts of generator.

5.4. Comparative analysis

To further investigate the consistency and effectiveness of proposed measure, the same numeric example interpreted by Radhika [32], Divsalar [27], Devadoss [1] and Dadzie [10]. Here, we assume same assumption and weight information, the ranking obtained by proposed as well as existing method are mentioned in Table 13. It reveals that proposed method is compatible with existing methods. The results obtained from proposed method shows the effectiveness in IF environment however the approach used in proposed method is totally different from the existing method which are mentioned in Table 13.

Table 13: Comparison with existing measures.

Methods	Ranking preference
Method suggested (Radhika [32])	$\varphi_2 > \varphi_4 > \varphi_1 > \varphi_3 > \varphi_5$
Method suggested (Divsalar [27])	$\varphi_3 > \varphi_1 > \varphi_4 > \varphi_2 > \varphi_5$
Method suggested (Devadoss [1])	$\varphi_4 > \varphi_1 > \varphi_2 > \varphi_3 > \varphi_5$
Method suggested (Dadzie [10])	$\varphi_5 > \varphi_2 > \varphi_2 > \varphi_4 > \varphi_3$
Proposed method	$\varphi_1 > \varphi_5 > \varphi_4 > \varphi_2 > \varphi_3$

In addition, we uses normalize criteria weights to find compatible and reliable information. Moreover, we also computed the objective criteria weight employing programming model. The key innovation of paper is the normalization of decision matrix by means of correlation coefficient based VIKOR approach. Therefore, we concluded that proposed method is more practical and easy to find ranking.

6. Conclusion

In this paper, a new entropy which is generalization of Shannon has been suggested for probabilistic distribution and intuitionistic fuzzy theory. Further, using the concept of correlation coefficient extended IF-VIKOR method has been introduced. Additionally, we discussed the application of proposed measure in MCDM problem using numerical example. Here, we discussed the two approaches for the determination of criteria weights that is partially known and unknown criteria weight. Further, we examine the sensitive analysis to measure the reliability of proposed method. Finally, comparison has been done with existing measure which shows the compatibility of proposed measure with existing one. In future, proposed work extended for interval valued IFSs, picture fuzzy set, Q-Orthopair fuzzy set etc.

Acknowledgements

The authors are thankful to the anonymous reviewers for their valuable suggestions and comments to improve this manuscript.

References

[1] A. Victor Devadoss (2017).A Hybrid Scenario FCM with VIKOR Technique for Ranking the factors, Vol.119, no.9, 233-244, ISSN;1314-3345.

[2] Aczel, J. and Darcozy, Z. (1975). On Measures of Information and Their Characterization, Academic Press, New York.

[3] Atanassov, K. T. (1986). *Intuitionistic fuzzy Sets*, Fuzzy Sets Syst, Vol.20, 87-96.

[4] Benayoun, R., Roy, B. and Sussman, B. (1966). *ELECTRE Une methode pour guider le choix en presence de points de vue multiples*, Note de travail 49. Direction Scientifique, SEMAMETRA International.

[5] Bhandari, D., Pal, N. R. (1975). *Some new information measures for fuzzy sets*, Inf. Sci., Vol.67, 209-228.

[6] Boran, F. E., Genc, S. and Akay, D. (2011). *Personal selection based on intuitionistic fuzzy sets*, Hum. Factors Ergon. Manuf. Serv. Ind., Vol.21, 493-503.

[7] Brans, J. P. and Mareschel, V. (1984). *Promethee: A new family of outranking methods in multicriteria analysis*, In J.P. brans (Ed.), Operational Research, Vol.84, 477-490.

[8] Chen, T. and Li, C. (2010). *Determining objective weights with intuitionistic fuzzy entropy measures: a Comparative analysis*, Inf. Sci., Vol.180, 4207-4222.

[9] De Luca, A. and Termini, S. (1972). *A definition of non-probabilistic entropy in the setting of fuzzy set theory*, Inf. Control, Vol.20, 301-312.

[10] Eric Afful-Dadzie (2014). Fuzzy VIKOR approach: Evaluating quality of internet health information. DOI:10.15439/2014F203

[11] Boran, F. E., Genc, S., Kurt, M. and Akay, D. (2009). *A multicriteria intuitionistic fuzzy group decision making for supplier selection with Topsis method*, Exp. Syst. Appl., Vol.36, No.8, 11363-11368.

[12] Gao, M. M., Sun, T. and Zhu, J. J. (2014). *Revised axiomatic definition and structural formula of intuitionistic fuzzy entropy*, Control. Decis., Vol.29, No.3, 470-474.

[13] Gerstenkorn, T. and Manko, J. (1991). *Correlation of intuitionistic fuzzy sets*, Fuzzy Sets Syst, Vol.44, 39-43.

[14] Guo, K. H. and Song, Q. (2014). *On the entropy for Atanassov's intuitionistic fuzzy Sets: an interpretation from the perspective of amount of knowledge*, Appl. Soft. Comput., Vol.24, 328-340.

[15] Liu, H. W. and Wang, G. J. (2007). *Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets*, Euro. J. Operat. Res., Vol.179, No.1, 220-233.

[16] Hwang, C. L. and Yoon, K. (1981). Multiple Attribute Decision Making, In: Lecture Notes in Economics and Mathematical Systems 186. Springer-Verlag, Berlin.

[17] Hung, W. L. and Yang, M. S. (2006). *Fuzzy entropy on intuitionistic fuzzy sets*, Int. J. Intell. Syst., Vol.21, No.4, 443-451.

[18] Joshi, R. and Kumar, S. (2016). *A new approach in multiple attribute decision making using R-norm entropy and Hamming distance measure*, International Journal of Information and Management Sciences, Vol.27, No.3, 253-268.

[19] Joshi, R. and Kumar, S. (2017). *Application of interval-valued intuitionistic fuzzy-norm entropy in multiple attribute decision making*, International Journal of Information and Management Sciences, Vol.28, No.3, 233-251.

[20] Joshi, R. and Kumar, S. (2018). *A new parametric intuitionistic fuzzy entropy and its application in multiple attribute decision making*, Int. J. Appl. Compute. Math., Vol.4, No.1, 52.

[21] Joshi, R. and Kumar, S. (2018). *A novel fuzzy decision making method using entropy weights based correlation coefficients under intuitionistic fuzzy environment*, International Journal of Fuzzy Systems, DOI 10.1007/s40815-018-0538-8.

[22] Joshi, R. and Kumar, S. (2019). *A new approach in multiple attribute decision making using exponential hesitant fuzzy entropy*, International Journal of Information and Management Sciences, Vol.30, No.4, 305-322.

[23] Li, H.-C., You, J.-X., You, X.-Y. and Shan, M.-M. (2015). *A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method*, Appl. Soft Comput., Vol.28, 579-588.

[24] Liu, M. F. and Ren, H. P. (2015). *A study of multi-attribute decision making based on a new intuitionistic fuzzy entropy measure*, Syst. Eng. Theor. Pract., Vol.35, No.11, 2909-2916.

[25] Lin, L., Yuan, X. H., Xia, Z. Q. (2007). *Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets*, J Comp. Syst. Sci., Vol.73, No.1, 84-88.

[26] Mao, J. J., Yao, D. B. and Wang, C. C. (2013). *A novel cross entropy and entropy measures of IFSs and their applications*, Knowl. Based Syst., Vol.48, 37-45.

[27] Mehdi Divsalar (2017). *Extension of the VIKOR method for group decision making with extentded hesitant fuzzy linguistic information*, DOI 10.1007/s00521-017-2944-5.

[28] Mishra, A. R., Rani, P. and Jain, D. (2017). *Information measures based topsis method for multi criteria decision making problem in intuitionistic fuzzy environment*, Iran. J. Fuzz. Syst., Vol.14, No.6, 41-63.

[29] Opricovic, S. and Tzeng, G. H. (2007). *Extended VIKOR method in comparison with outranking methods*, European Journal of Operational Research, Vol.178, 514-529.

[30] Opricovic, S. (1998). Multi-criteria optimization of civil engineering systems, Ph.D. Thesis, University of Belgrade.

[31] Renyi, A. (1961). *On measures of entropy and information*, In: Proceeding of 4th Barkley Symposium on Mathematical Statistics and probability, Vol.1, 547, University of California Press.

[32] Radhika, S. and Sammeer Kumar, D. (2017). *VIKOR method for multicriteria decision making in Academic staff selection*, Journal of production Research and management, Vol.3, issue 2, ISSN:2249-4766.

[33] Shannon, C. E. (1948). *A mathematical theory of communication*, Bell Syst. Tech. J., Vol.27, 379-423.

[34] Szmidt, E. and Kacprzyk, J. (2002). *Using intuitionistic fuzzy sets in group decision-making*, Control Cybern., Vol.31, 1037-1054.

[35] Verma R. K. and Sharma, B. D. (2014). *On intuitionistic fuzzy entropy of order-α*, Adv. Fuzzy Syst., Article ID 789890, 8 pages.

[36] Vlachos, I. K. and Sergiadis, G. D. (2007). *Intuitionistic fuzzy information- application to pattern recognition*, Pattern Recognit. Lett., Vol.28, 197-206.

[37] Vikas, A. and Kumar, S. (2020). *Fuzzy entropy measures with an applications in decision making under bipolar fuzzy environment based on topsis method*, International Journal of Information and Management Sciences, Vol.31, 99-121.

[38] Wang J. and Wang P. (2012). *Intuitionistic linguistic fuzzy multi criteria decision-Making method based on intuitionistic fuzzy entropy*, Control Decis., Vol.27, 1694-1698.

[39] Xia, M. and Xu, Z. S. (2012). *Entropy/Cross entropy based group decision making under intuitionistic fuzzy environment*, Inf. Fusion, Vol.13, 31-47.

[40] Xiao Luo and Xuanzi Wang (2017). *Extended VIKOR method for intuitionistic fuzzy multiattribute decision-making based on a new distance measure*, Hindawi, Mathematical problems in Engineering; article ID 4072486, 16 pages.

[41] Xiong, S. H., Wu, S., Chen, Z. S., Li, Y. L. (2017). *Generalized intuitionistic fuzzy entropy and its application in weight determination*, Control Decis., Vol.329, No.5, 845-854.

[42] Xu, Z. S. and Yager, R. R. (2006). *Some geometric aggregation operators based on intuitionistic fuzzy sets*, Int. J. gen. Syst., Vol.35, 417-433.

[43] Xu, Z. S. (2007). *Intuitionistic fuzzy aggregation operators*, IEEE Trans. Fuzzy Systs., Vol.15, 1179-1187.

[44] Xu, Z. S. and Hu, H. (2010). *Projection models for intuitionistic fuzzy multiple attribute decision making*, Int. J. Inf. Technol. Decis. Mak., Vol.9, 267-280.

[45] Ye, J. (2010). *Fuzzy decision making method based on the weighted correlation coefficient under intuitionistic fuzzy environment*, European Journal of Operational Research, Vol.205, 202-204.

[46] Zadeh, L. A. (1965). *Fuzzy sets*, Information and Computation, Vol.8, No.3, 338-353.

[47] Zadeh, L. A. (1968). *Probability measures of fuzzy events*, J. Math. Anal. Appl., Vol.23, 421-427.

[48] Zadeh, L. A. (1975). *The concept of linguistic variable and its application to approximate reasoning*, I. Inf. Sci., Vol.8, 199-249.

[49] Zhu, Y. J. and Li, D. F. (2016). *A new definition and formula of entropy for intuitionistic fuzzy sets*, J. Intell. Fuzzy. Syst., Vol.30, No.6, 3057-3066.

Department of Mathematics, Maharishi Markandeshwar (Deemed to be University).

E-mail: rakhigupta.k@gmail.com

Major area (s): Information Theory, Fuzzy Information Measures, coding theory, Divergence and similarity measures.

Department of Mathematics, Maharishi Markandeshwar (Deemed to be University).

E-mail: profsatish74@mmumullana.org

Major area (s): Information theory, measure theory, fuzzy information measures, coding theory and decision-making problems.

(Received ??? 202?; accepted ??? 202?)