

## GM(1,1) Modeling of Failure Rate Prediction for Preventive Maintenance

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### Abstract

In preventive maintenance, the prediction of failure rate is of great significance for making accurate and reasonable maintenance plan, mastering the initiative of maintenance and giving full play to the use efficiency of equipment. The traditional failure rate methods and models rely on a large amount of statistics. However, statistics data are often unavailable, which creates difficulties in predicting failure rates. Aiming at the current situation of “little data, poor information”, this paper summarizes existing research on failure rate prediction methods at home and abroad, and the  $GM(1,1)$  model is used to predict the failure rate. This article analyzes the concrete examples of cranes. Firstly, the  $GM(1,1)$  model and the discrete  $GM(1,1)$  model are established based on the determination of the length of the model for the fault rate curve with large irregular fluctuation, and the models is compared and optimized. Secondly, the linear regression  $GM(1,1)$  model is introduced for the curve with linear trend, and three models are compared and optimized. It has practical application value in the prediction of equipment failure rate.

*Keywords:* Failure rate forecast,  $GM(1,1)$  model, discrete  $GM(1,1)$  model, linear regression  $GM(1,1)$  model.

### 1. Introduction

Bridge cranes are bulky, have frequent cargoes transportation and a large number of relevant operators during operation. In the event of an accident, the lighter affects the progress of the work, and the heavy ones cause irreparable damage to the people's lives and state property. In order to improve this situation, the state has formulated a number of standards for the design, manufacture, installation and use of lifting machinery. It also strengthened the management of lifting machinery, but the lifting of chemical weapons accidents remains high. In 2016, the General Administration of Quality Supervision, Inspection and Quarantine reported the safety situation of special equipment in China. There were 283 special equipment accidents in the country, with 282 deaths and 330 injuries. Among them, the number of electric crane accidents accounted for 21.91%, and the death toll accounted for 34.75%. In the case of crane accidents, there were

35 accidents caused by “three violations” and 9 accidents caused by hidden equipment. Although cranes are inspected and repaired regularly, the hidden dangers in operation easily lead to the occurrence of faults, which can lead to accidents. Therefore, a reasonable analysis and summary of the fault is of great significance to prevent the occurrence of the fault and to prevent the occurrence of the accident.

With the development of system automation and complication, people pay more and more attention to the research of equipment maintenance, such as computer networks, military systems and industrial systems maintenance (see Gao [2]). It is very important to predict the failure rate accurately. On the one hand, it can prevent accidents in advance, minimize equipment failure time and ensure the normal operation of equipment. On the other hand, it can instruct purchase of spare parts for necessary preventive maintenance of equipment to avoid unnecessary waste and the lack of reserves so that we can promptly deal with failure. Therefore, the prediction of failure rate is particularly important, and the demand for failure rate prediction is getting higher and higher. There are many methods and models for predicting the failure rate at home and abroad (see Lin [7]).

The function of the prediction is to analyze the data by building a model, or by evaluating the range of characteristic values of the unlabeled sample. Forecasting is a way to summarize and describe the trends of upcoming events or data. Researchers all around the world have put forward some effective methods, including regression analysis method, time series analysis method, gray analysis method neural network and support vector machine method (see Wang [16]). Regression analysis is a quantitative method to establish the regression analysis model from the correlation between the predicted object and its dependent variable to predict the future development trend of the object. Regression model is the simplest prediction model. The advantage of the regression model prediction are easy operation and simple method. But the accuracy of prediction is not high, and it needs a large number of data samples (see Lin [8]). The time series forecasting method selects appropriate models and parameters to establish the forecasting model and predicts the unknown data attributes by using the information of data change characteristics in given time series. The time series forecast method generally requires the data that have the better stability and the accuracy of prediction is not high. This method requires data may not have the mutation and a large amount of historical data (see Jiang [4]). From the above analysis, we can find that the above methods of failure rate prediction are not suitable for the prediction of the failure rate in the case of “little data” and “poor information” (see Zhang [21]).

Many failure rate prediction methods rely on a large number of statistical data to estimate the distribution of various parameters, and then predict the failure rate. However, sometimes it is not easy to obtain statistics data so that there will be some difficulties in predicting the failure rate. In order to solve the prediction of failure rate in the condition of “little data and poor information”, this paper introduces the gray theory to predict the failure rate. Firstly, the modeling length is discussed in the gray theory prediction. Secondly, according to different failure rate characteristics, the improved method of  $GM(1, 1)$  model, the discrete  $GM(1, 1)$  model and linear regression  $GM(1, 1)$  model are introduced. Finally, the improved methods are compared in specific cases to find a more accurate method for the failure rate.

**Remark 1.** To improve the readability, the descriptions of the following notation have been used in this research work:

- $X^{(0)}$  : is non negative original sequence
- $X^{(1)}(t)$  : is resulting sequence
- $Z^{(0)}$  : is the immediate neighbor
- $z^{(1)}$  : is the sequence of immediately adjacent means of  $x(1)$
- $a$  : is development factor
- $b$  : is action amount
- $\hat{x}(t + 1)$  : is the time response function
- $\beta_1\beta_2$  : is the parameter of least square method
- $B$  : is the parameter estimation matrix
- $c_1, c_2, c_3$  : is undetermined parameter
- $\nu$  : is undetermined parameter
- $\tilde{V}$  : is the approximate Solution of  $\nu$
- $Z(t)$  : is the set parameter sequence
- $\tilde{V}_m(t)$  : is the different m values of  $\tilde{V}$
- $\hat{V}$  : is the average of  $\nu$
- $C$  : is the parameter matrix
- $A$  : is the parameter estimation matrix
- $\hat{x}^{(1)}(t)$  : is the generated sequence
- $\hat{x}^{(0)}(t + 1)$  : is the sequence reduced reduction.

## 2. Prediction Model for Little Data and Poor Information

### 2.1. GM(1, 1) Model

As a new discipline, gray theory's structural system has already been set up after more than 20 years of continuous development and in-depth research. The gray forecasting system marked by the GM(1, 1) model is one of the developed theories in the gray theory system. The gray model of GM(1, 1) means 1-order and 1-variable. The gray model of GM(1, 1) is defined as the gray model of the system with "small sample" and "poor information". The uni variate constant coefficient differential equation is used to describe a generalized energy system, so that the GM(1, 1) model becomes a gray model suitable for forecasting (see Zhao [20]).

**Theorem 1.** Assume  $X^{(0)}$  is a non-negative original sequence, then the result of the GM(1, 1) prediction model is  $\hat{x}^{(0)}(t + 1) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t)$ ,  $t = 1, 2, \dots, n$ .

**Proof.** We follow the steps below to verify.

(1) Let  $X^{(0)}$  be a non-negative original sequence:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)). \quad (2.1)$$

Accumulate the original sequence  $X^{(0)}$  once to obtain a new data sequence:

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \quad (2.2)$$

where  $x^{(1)}(t) = \sum_{i=1}^t x^{(0)}(i)$ ,  $(t = 1, 2, \dots, n)$ .

(2) The resulting sequence  $X^{(1)}(t)$  has the following first-order linear whitening differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \quad (2.3)$$

(3) To make the sequence of cumulative additions smoother, make the immediate neighbor mean:

$$Z^{(0)} = (z^{(1)}(2), z^{(1)}(3), z^{(1)}(4), \dots, z^{(1)}(n)) \quad (2.4)$$

where  $z^{(1)}(t) = 0.5(x^{(1)}(t) + x^{(1)}(t-1))$ .

The basic form of the  $GM(1,1)$  model thus obtained is:

$$x^{(0)}(t) + az^{(1)}(t) = b \quad (2.5)$$

where “ $a$ ” is development factor; “ $b$ ” is ash action amount;  $z^{(1)}$  is the sequence of immediately adjacent means of  $x^{(1)}$ .

(4) We solve the basic form parameter  $a, b$  of  $GM(1,1)$  model and estimate the values of “ $a$ ” and “ $b$ ” by the least square method:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \quad (2.6)$$

where  $B = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ M & M \\ -Z^{(1)}(n) & 1 \end{bmatrix}$ ,  $Y = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T$ .

(5) The values of  $a$  and  $b$  are substituted into the  $GM(1,1)$  model of the whitening equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \quad (2.7)$$

We can get the time response function of  $GM(1,1)$  model as:

$$\hat{x}(t+1) = (x^{(0)}(1) - \frac{b}{a})e^{-at} + \frac{b}{a}, \quad t = 1, 2, \dots, n-1. \quad (2.8)$$

We can write its form as:

$$\hat{x}(t+1) = c_1 e^{-ut} + c_2. \quad (2.9)$$

(6) Reduced reduction:

$$\hat{x}^{(0)}(t + 1) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t), \quad t = 1, 2, \dots, n. \tag{2.10}$$

**2.2. Discrete GM(1, 1) model based on simulated exponential sequence**

The traditional GM(1, 1) simulation exponential growth sequence has a big error, and the discrete GM(1, 1) model can accurately simulate the exponential sequences. Therefore, we consider the discrete GM(1, 1) model to predict the failure rate of equipment (see Xu [18]).

**Theorem 2.** Assume  $X^{(0)}$  is a non-negative original sequence, then the result of the discrete GM(1, 1) prediction model is  $\hat{x}^{(1)}(t + 1) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t)$ .

**Proof.** We follow the steps below to verify.

(1) Let  $X^{(0)}$  be a non-negative original sequence and the original sequence of  $X^{(0)}$  to generate a cumulative. The new data column to establish a discrete GM(1, 1) model is:

$$x^{(1)}(t + 1) = b_1 x^{(1)}(t) + b_2. \tag{2.11}$$

(2) Estimating  $\beta_1$  and  $\beta_2$  in a discrete GM(1, 1) model using the least-squares method yields:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = (B^T B)^{-1} B^T Y \tag{2.12}$$

where  $B = \begin{bmatrix} -X^{(1)}(1) & 1 \\ -X^{(1)}(2) & 1 \\ M & M \\ -X^{(1)}(n - 1) & 1 \end{bmatrix},$

$$Y = (x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n))^T. \tag{2.13}$$

(3) According to  $x^{(1)}(1) = x^{(0)}(1)$ , obtain the recursive function of the discrete GM(1, 1) model:

$$\hat{x}(t + 1) = \beta_1^t \times x^{(0)}(1) + \frac{1 - \beta_1^t}{1 - \beta_1} \times \beta_2, \quad (t = 1, 2, \dots, n). \tag{2.14}$$

Substituting  $\beta_1$  and  $\beta_2$  into the recursive function of the GM(1, 1) model yields  $\hat{x}^{(1)}(t + 1)$ ;

(4) Reduced reduction:

$$\hat{x}^{(1)}(t + 1) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t). \tag{2.15}$$

**2.3. Linear regression GM(1, 1) model based on comprehensive trend sequence**

Due to the complexity of the equipment and the environment, there is a great difference in the law of failure. Therefore, “the bathtub curve” does not applicable to all equipment. In order to predict the failure rate of both linear and exponential trends, we consider the linear GM(1, 1) model to predict the failure rate of the equipment (see Chen [1], Shao [11], Shao [12], Shao [13]). The process is as follows:

**Theorem 3.** Assume  $X^{(1)}(t)$  is a use linear regression equation and the time-dependent function of the GM(1,1) model to fit the accumulative generation sequence, and then the result of the discrete GM(1,1) prediction model is  $\hat{x}^{(0)}(t+1) = \hat{x}^{(1)}(t+1) - \hat{x}^{(1)}(t)$ ,  $t = 1, 2, \dots, n$ .

**Proof.** We follow the steps below to verify.

- (1) Use linear regression equation and the time-dependent function of the GM(1,1) model to fit the accumulative generation sequence. So write the generated sequence:

$$\hat{x}^{(1)}(t) = C_1 e^{vt} + C_2 t + C_3 \quad (2.16)$$

where  $C_1, C_2, C_3$  and  $v$  are undetermined parameter.

- (2) To determine the above parameters, set parameter sequence:

$$Z(t) = C_1 \exp(vt)[\exp(v) - 1] + C_2 \quad (2.17)$$

where  $t = 1, 2, \dots, n - 1$ .

- (3) Let  $Y_m(t) = Z(t+m) - Z(t)$ , then

$$\begin{aligned} Y_m(t) &= C_1 \exp(vt)[\exp(v_m) - 1][\exp(v) - 1] \\ Y_m(t+1) &= C_1 \exp(v(t+1))[\exp(v_m) - 1] \\ &\quad [\exp(v) - 1][\exp(v) - 1] \end{aligned} \quad (2.18)$$

where  $m = 1, 2, \dots, n - 3$ ;  $t = 1, 2, \dots, n - m - 2$ . The solution of the  $v$  obtained by comparing the above two expressions are:

$$v = \ln[Y_m(t+1)/Y_m(t)]. \quad (2.19)$$

- (4) Change the formula  $\hat{x}^{(1)}$  into  $x^{(1)}$ , get the approximate solution  $\tilde{V}$ . Different values of  $m$ , different values of  $\tilde{V}$ . Make their average value as the estimation of  $v$ .
- (5) Calculate  $\tilde{V}_m(t)$ :

$$\tilde{V}_m(t) = \ln[Y_m(t+1)/Y_m(t)]. \quad (2.20)$$

The number of  $\tilde{V}$  is  $(n-2)(n-3)/2$ , and one can get the value  $\tilde{V}$ :

$$\tilde{V} = \frac{\sum_{m=1}^{n-3} \sum_{t=1}^{n-m-2} \tilde{V}_m(t)}{(n-2)(n-3)/2}. \quad (2.21)$$

- (6) Let  $L(t) = e^{\tilde{V}t}$

$$\hat{x}^{(1)} = c_1 L(t) + c_2 t + c_3. \quad (2.22)$$

- (7) Obtained by the least square method:

$$C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = (A^T A)^{-1} A^T X^{(1)} \quad (2.23)$$

where  $A = \begin{bmatrix} L(1) & 1 & 1 \\ L(2) & 2 & 1 \\ M & M & M \\ L(n) & n & 1 \end{bmatrix}$ ,  $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))^T$ .

Obtain the predicted value of the generated sequence:

$$\hat{x}^{(1)}(t) = c_1 e^{\hat{v}t} + c_2 t + c_3. \tag{2.24}$$

(8) Reduced reduction:

$$\hat{x}^{(0)}(t + 1) = \hat{x}^{(1)}(t + 1) - \hat{x}^{(1)}(t), \quad t = 1, 2, \dots, n. \tag{2.25}$$

### 3. Raising Questions about the Crane Failure

The bridge crane is the irreplaceable equipment for modernization and mechanization in the process of industrialization. Therefore, the bridge crane is widely used in the machinery manufacturing, steel production units, the port logistics and transportation departments (see Song [15]). The bridge cranes mainly has three different structure types: ordinary bridge crane, simple beam bridge crane and metallurgical especial bridge crane. The ordinary bridge crane mainly includes the crab, the structure frame and the operating mechanism (see Hu [3]). The structure of the ordinary bridge crane is shown in Figure 1.

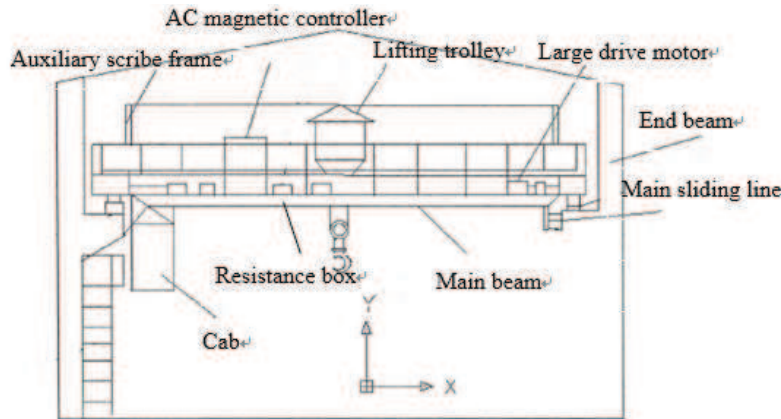


Figure 1: Diagram of the structure of the bridge crane.

In the case of bridge crane failure, the bridge crane causes frequent accidents due to lack of measures or lack of effective management. It results in casualties and property losses. Therefore, it is particularly important to find appropriate method to analyze and even predict the failure of the bridge crane (see Sai [10]).

This paper only considers the main faults types of bridge cranes, and mainly considers the location of faults from the structure of bridge cranes. Through the analysis of the bridge cranes fault, it is determined that the main fault modes of the bridge crane

Table 1: the main fault of bridge crane.

Fault Type	Specific fault
Rail Deformation	Trolley slipping; Inclined walking of trolley; Wear of wheel; Rail-gnawing
Fault of the main girder	Subsidence of the main girder; Bending of the main girder
Fault of lifting attachments	Groove wear of sheave; Wear and fracture of wire rope; Wear and displacement of the hook; Fracture of the drum
Brake failure	Slipping of the break; Temperature rise of brake; Failure of internal contracting brake
Fault of the speed reducer	Abnormal Vibration; Oil-leakage in gearbox; Tooth surface abrasion
Fault of electric system	Wear of the motor; Strain of the motor; Short circuit

are five types: metal structural faults, faults of lifting attachments, brake faults, speed reducer faults, and electrical system faults (see Li [6], Xiao [17]). The specific faults are shown in Table 1.

#### 4. Failure Rate Prediction about the Cranes

The repair maintenance tasks of a certain type of crane are all taken care of by the unit maintenance support center. All spare parts are repaired and replaced through the maintenance support center. We can know the failure rate of a certain type of crane through checking detailed data from the maintenance support center of the user (see Sheu [14], Zhao [19]).

##### 4.1. Data Acquisition and Failure Rate Analysis

The failure rate of the certain type of crane in the wear-out period when it is working normally is shown in Table 2. In this paper, we use the  $GM(1, 1)$  model introduced above to predict the failure rate, where “ $t$ ” is the working time of the crane (unit: month),  $\lambda(t)$  is the failure rate (unit:  $10^{-5}$  times/h).

In this paper, through analyzing the loss rate of the bearing of the crane in 48 months we can get the characteristics of the loss rate as shown in Figure 2.

From the figure, we can see that the failure rate of the bearing is a random data with relatively large volatility and the curve is non-linear without laws. We can also find that its failure rate gradually decreases because of the improvement of the technology with to the passage of time. Grey forecasting model is suitable for the prediction of small sample and poor information. Even more, we can get more uniform data format and more results that are accurate from the data accumulated generating. There is a great



Table 2: The Failure rate of the crane during the loss period.

$t$	$\lambda(t)$	$t$	$\lambda(t)$	$t$	$\lambda(t)$	$t$	$\lambda(t)$
1	10.73	13	11.45	25	10.14	37	10.03
2	10.04	14	11.21	26	11.18	38	10.73
3	11.39	15	10.55	27	10.58	39	11.98
4	10.98	16	10.42	28	8.95	40	9.72
5	10.60	17	10.06	29	9.45	41	10.29
6	9.69	18	9.83	30	8.93	42	11.6
7	10.03	19	10.25	31	9.66	43	9.14
8	10.11	20	11.02	32	10.01	44	8.18
9	9.85	21	9.93	33	10.33	45	7.59
10	10.17	22	9.76	34	11.43	46	7.27
11	11.04	23	9.81	35	10.32	47	9.38
12	10.23	24	10.29	36	10.62	48	8.92

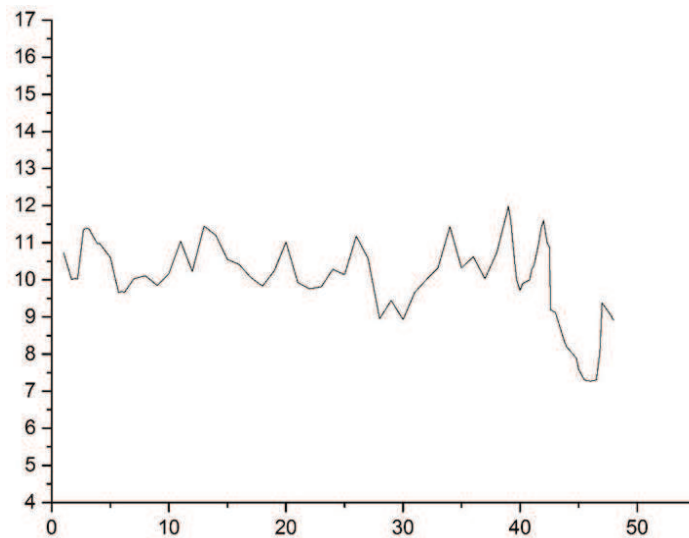


Figure 2: Loss rate curve of crane bearing.

error when using traditional  $GM(1, 1)$  model to simulate exponential growth sequence but the discrete  $GM(1, 1)$  model can do it accurately. Therefore, we consider predicting the failure rate of the equipment in the loss period by the discrete  $GM(1, 1)$  model.

When establishing the  $GM(1, 1)$  prediction model, we divide the sample data into

Table 3: Grouping of  $GM(1, 1)$  models.

Number	Enter the sample	Expected output
1	$x(1), x(2), \dots, x(m)$	$x(m + 1)$
2	$x(2), x(3), \dots, x(m + 1)$	$x(m + 2)$
$\vdots$	$\vdots$	$\vdots$
$n$	$x(n - m), x(n - m + 1), \dots, x(n - 1)$	$x(n)$

Table 4: Results from different modeling sequence length prediction.

Number	Actual value	$GM(1, 1)$ modeling sequence length					
		15		12		9	
		Predictive value	Relative error	Predictive value	Relative error	Predictive value	Relative error
45	7.59	6.89	0.0922	6.03	0.2055	5.96	0.2147
46	7.27	6.85	0.0578	6.53	0.1018	6.55	0.2147
47	9.38	8.63	0.0799	8.85	0.0565	8.26	0.1194
48	8.92	9.14	0.0247	8.32	0.0672	8.18	0.0830

two parts. The failure rate of the previous period is taken as the initial sample, and the  $GM(1, 1)$  model is built with these data respectively (see Liu [9]). Because the prediction of the  $GM(1, 1)$  model is related to the selected model length, the paper selected 15, 12, 9 three kinds of length to build model and compared which model and length prediction are good. Based on the first  $n$  data as input to the  $GM(1, 1)$  model, we can predict the  $(n + 1)$  result, and the samples are shown in Table 3 (see Jaleel [5]).

According to the above forecasting process, we take different modeling lengths of 15, 12 and 9 to predict the bearing failure rate in the last 4 months respectively. You can see the results we obtained in Table 4.

In order to clearly see the impact of the three different length of the modeling length on the prediction results, we have fitted the prediction of the three predictions, as shown in Figure 3.

As can be seen from the figure above, when the length of modeling is 15, the prediction result is the closest to the true value. Therefore, this article selects the modeling length of 15. When we predict the completion of a value, we remove a value from the beginning, and then add a new predicted value later, and then continue to predict the next data, until all four data measured repeatedly. We can get the crane system  $GM(1, 1)$  model and discrete  $GM(1, 1)$  model trend chart in Figure 4 and the error table in Table 5.

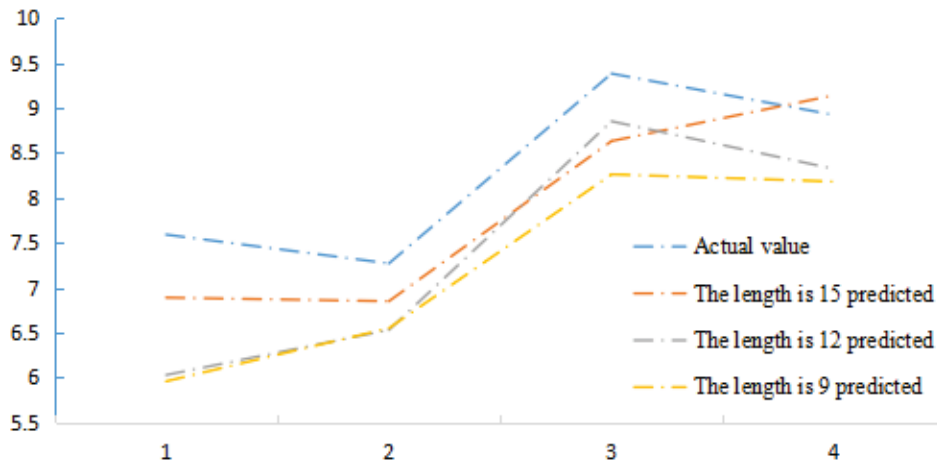


Figure 3: Comparison of prediction results for different modeling sequences of  $GM(1, 1)$ .

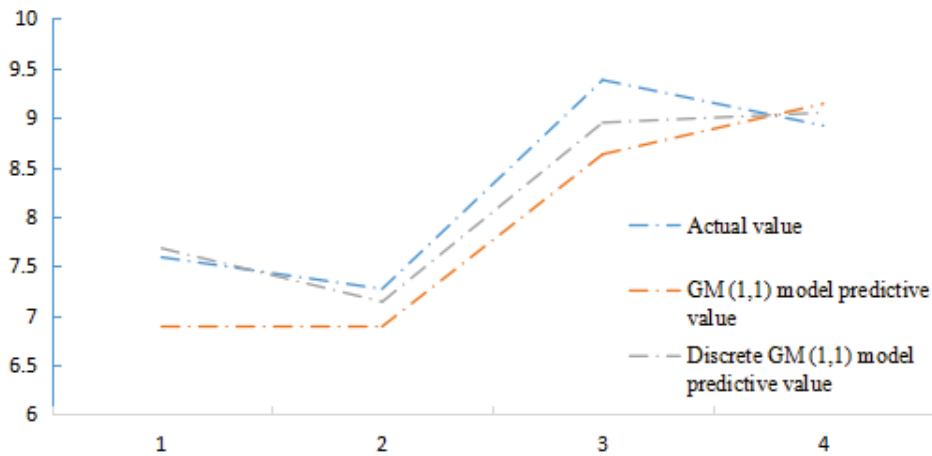


Figure 4:  $GM(1, 1)$  model and discrete  $GM(1, 1)$  model fitting trend.

Table 5:  $GM(1, 1)$  model and discrete  $GM(1, 1)$  model error test table.

Number	Actual value	$GM(1, 1)$ model			Discrete $GM(1, 1)$ model		
		Predictive value	Relative error	Average value	Predictive relative error	Relative value	Average relative error
45	7.59	6.89	0.922	0.2711	7.68	0.012	0.023
46	7.27	6.85	0.0578		7.14	0.019	
47	9.38	8.63	0.0799		8.95	0.046	
48	8.92	9.14	0.0247		9.05	0.015	

By comparing the results of the average relative error in the above examples, we can see that although both methods can predict the failure rate, the discrete gray  $GM(1, 1)$  model improves the prediction accuracy of typical failure rate compared with the traditional gray  $GM(1, 1)$  model.

**4.2. Failure rate prediction based on linear  $GM(1, 1)$  combination model**

Due to the complexity of the equipment and the use of different environments, there is a difference in the failure law, so bathtub curve does not apply to all equipment. General equipment can use these six kinds of curves to describe the basic type of failure rate curve  $\lambda(t)$  shown in Figure 5.

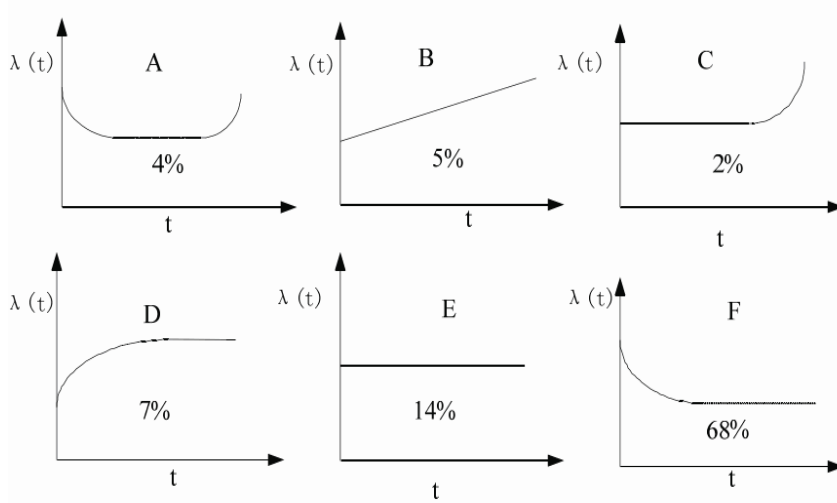


Figure 5: Basic types of failure rate curves.

Through the analysis of the above failure rate curve, we find that the failure rate obeys the exponential distribution and the linear distribution in different stages. Aiming at the problem that the original data of failure rate is relatively few, we consider using gray system model to predict. It is difficult to describe other trends of sequence data. Therefore, in order to predict the failure rates with both linear trends and exponential trends, the linear regression  $GM(1, 1)$  model is considered to predict the failure rate of equipment. It can improve the lack of linear factors in the  $GM(1, 1)$  model and the lack of exponential growth in the linear regression model. Using linear regression equation and exponential equation to fit the failure rate curve can give full play to the advantages of gray data-less modeling and regression model and improve the prediction accuracy.

Through the data review of a crane, we can know the failure rate of a crane. The failure rate of a crane in normal use in the past 9 years is shown in Table 6, where “ $t$ ” is the working hours of a crane (unit: year),  $\lambda(t)$  is a crane failure rate (unit :  $10^{-6}$  times/h).

Table 6: Crane wear-out failure rate.

Year( $t$ )	1	2	3	4	5	6	7	8	9
Failure rate $\lambda(t)$	3.4	3.5	3.9	4	4.1	4.4	4.7	5.3	5.7

Table 7: Three types of model error test table.

Years	Actual value	Linear regression model		$GM(1, 1)$ model		Linear Regression $GM(1, 1)$ Model	
		Predictive value	Relative error	Predictive value	Relative error	Predictive value	Relative error
2	3.5	3.41	2.5	3.31	5.28	3.49	0.28
3	3.9	3.8	2.56	3.69	5.38	3.85	1.28
4	4	4.08	2	3.99	0.25	3.97	0.75
5	4.1	4.32	5.4	4.27	4.1	4.14	1
6	4.4	4.73	7.5	4.74	7.7	4.53	3
7	4.7	4.94	5.1	5.04	7.2	4.74	0.9
8	5.3	5.21	1.7	5.35	0.9	5.34	0.7
9	5.7	5.3	7	5.68	0.16	5.66	0.34
Average relative error		4.22		3.87		1.03	

As seen from Table 6, the failure rate of cranes shows an increasing trend with the increase of service life, but there is obvious fluctuation and does not exactly conform to the exponential distribution. Therefore, the linear regression  $GM(1, 1)$  model is built to predict the failure rate. The forecast trends of the three models are shown in Figure 6 and the error-checking table in Table 7.

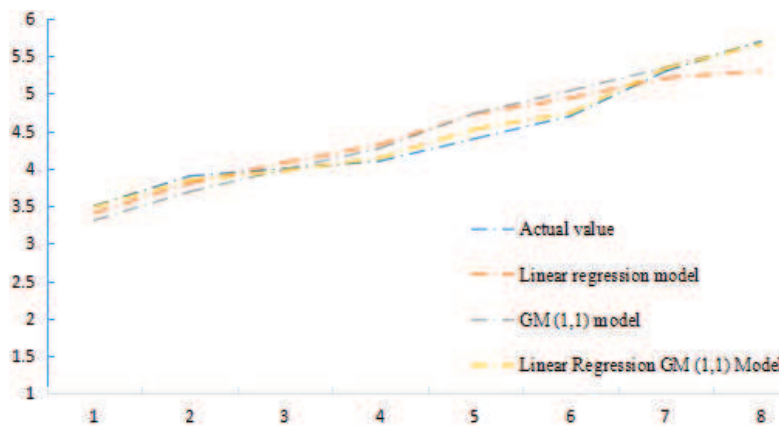


Figure 6: Three models predict fitting trend.

Through the comparison of the results of the average relative error in the above examples and the fitting of the predicted values, we can know that all three models can predict the failure rate. However, linear regression  $GM(1, 1)$  model can take advantage of the few data-requirement and regression factors related. It can synthesize many kinds of information, such as linear and exponential. Besides, the regularities of distribution may not be typically for the data.

Therefore, the linear regression  $GM(1,1)$  model is superior to the single  $GM(1,1)$  model and linear regression model in the accuracy. It has some practicality in the prediction of equipment failure rate.

## 5. Conclusion

In order to solve the problem of “failure of data and poor information”, the gray theory is proposed to predict the failure rate. The main research results of this paper are as followed:

In this paper, we use the  $GM(1,1)$  model to predict the failure rate of unconventional and volatility data, and discuss the modeling length. When the modeling length is 15, the prediction effect is the best. Therefore, the  $GM(1,1)$  modeling in this paper is to select the modeling length to be 15. In this paper, we introduce the discrete  $GM(1,1)$  model to predict the failure rate. As can be seen from the comparison of forecast results, the relative error and the average relative error of the discrete  $GM(1,1)$  model are less than the traditional  $GM(1,1)$  model, so the discrete  $GM(1,1)$  model is better than the  $GM(1,1)$  model prediction.

In this paper, by comparing the linear regression model,  $GM(1,1)$  model and the linear regression  $GM(1,1)$  model can be seen from the forecast results, the linear regression  $GM(1,1)$  model has the lowest relative error and the smallest average relative error, and the best prediction result. Therefore, we should use the linear regression  $GM(1,1)$  model when dealing with this type of failure rate.

With the accurate prediction of the failure rate, the spare parts storage can be effectively guided and maintenance support costs can be reduced. It also improves equipment availability and mission success rates. Accurately predicting the failure rate is of great importance in avoiding losses due to failures.

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