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Optimal Inventory-allocation Integrated Model for Perishable Items with Stochastic Demand in a Single-vendor Multi-retailer Supply Chain

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Abstract

This study investigates an increasingly widespread supply chain system that involves a vendor cyclically supplying specific perishable item to multiple retailers. In general, the vendor has to earlier and accurately determine inventory quantity to meet aggregate demand, and exactly allocate the inventory among a number of retailers to reduce the adjustment costs. Meanwhile, a vital challenge faced for vendors is to develop an efficient inventory-allocation decision model for given perishable item with stochastic and correlated retailer demands during upcoming selling period. To this end, an effective and practical analytical approach, which extends the newsvendor model to incorporate the considered inventory-allocation decision, is proposed here to simultaneously solve the optimal inventory quantity and allocation policy for maximizing expected vendor profits of perishable items. Especially, the lognormal distribution and Ito process is used here to model the behaviors of individual demand shift. Also, an effective integrated approach is presented to work out the aggregate demand during next selling period. Finally, numerical experiments are conducted to demonstrate and validate the proposed model and extract the valuable managerial findings.

Keywords: Inventory-allocation model, single-vendor multi-retailer supply chain, newsvendor model, perishable item, stochastic demand.

1. Introduction

The inventory policies in traditional supply chain systems are mostly dominated and determined by downstream retailers that are motivated for maximizing their profits. As a result, most studies on supply chain management and inventory control focused on investigating the economic ordering model from buyer's perspective. However, recently some business patterns activating an emerging tendency in supply chains is for the vendor to manage inventory such that the supplier or manufacturer acts as a leader and trigger of the supply chain and gets a predominant position from the retailers to help optimize wholesales. Among others, two typical systems are as follows: (1) Vendor concurrently serves as retailer and owns the retail stores. (2) In a consignment stock supply chain retailers cooperate with vendor and simply provide a selling site and warehouse in order for hedging the risk of sales, earn sales commission, and possibly receive compensation for inventory costs. In summary, this study looks at inventory decision of vendor in a supply chain and would be applicable to most vendor-managed inventory practices.

Besides vendor-managed inventory, the study considers and analyzes a common two-echelon supply chain system, which consists of a single vendor and multiple retailers and involves periodic supply and sale of given perishable item. Furthermore, vendor produces and delivers period by period a quantity corresponding to expected aggregate demand for given perishable item during the upcoming selling period and suitably allocate the inventory among multiple retailers. In practice, perishable items are common and necessary in everyday life, such as electronic components, fashion items, foodstuffs, beverages, pharmaceuticals, chemicals, printing goods, and so on. A crucial challenge for vendors in making an accurate and profitable inventory-allocation decision is subject to numerous volatile demands sourced from individual retailer and correlated demands between any two retailers during a selling period. If the accurate inventory-allocation decision is made by vendor, implying that the loss of unsold inventory, inventory expenses, shortage cost, and adjustment cost can be jointly reduced, and thus being expected to sizably improve vendor profitability and customer service level. Correspondingly, this study aims for developing a practical single-period inventory-allocation decision model for vendor of given perishable item in a single-vendor multi-retailer supply chain.

To this end, the production/delivery quantity and allocation of the given perishable item meets the real demand of each retailer in the supply chain as closely as possible. Furthermore, precise estimates of aggregate demand and individual demands derived from retailers become critical when unsold inventory remains little salvage value and adjustment costs are costly among retailers. Therein, adjustment cost represents a cost incurred by the shunting operation that delivers the item of surplus retailer to shortage retailer.

Inventory issues involving perishable items, which are characterized by over a finite selling horizon, is considered frequently as newsvendor problems and completely perishable and deteriorating inventory issues have received significant attention and been widely investigated in literature. Several representative researches on tackling survey of literature on perishable/deteriorating inventory rendered the overview of development trend. These contributions mainly were made by Nahmias [35], Raafat [36], Goyal and Giri [17], Khanlarzade et al. [29], and Janssen et al. [27]. Based on the works of their researches, the investigating issues for perishable/deteriorating inventory can been approximately generalized into a number of central study fields including, for example, review system, replenishment cycle, pricing policy, newsvendor and EOQ model, inventory management with a deteriorating rate, deterministic/probabilistic demand, perishable/deteriorating items with a fixed or random lifetime, returns policies, single product and multiple products, and single-period and multiple period ordering model. Among these fields, this study can be positioned as a single-period newsvendor model for a single perishable item with probabilistic demand and fixed lifetime.

Typical researches on single-vendor multi-buyer supply chain models include as follows. Ingene and Parry [26] and Chen et al. [12] put forward coordination mechanisms for a distribution system involving one supplier and multiple retailers to optimize channel-wide profits. Kim et al. [30] proposed an analytical model to integrate and synchronize raw material procurement, production of multiple items, and delivery to multiple retailers. Siajadi et al. [38] introduced a new methodology to obtain the joint economic lot size for a single-vendor multi-retailer problem. Chan and Kingsman [8] proposed a coordinated single-vendor multi-buyer supply chain model by synchronizing delivery and production cycles. Thangam and Uthayakumar [42] designed an approximate cost function through which a single-supplier and multiple-retailer supply chain can identify optimal reorder points. Chen and Chang [9] aimed to jointly determine the optimal retail price, replenishment cycle, and shipment number for deteriorating items in a one-manufacturer and multi-retailer channel. Hoque [20, 21, 22] studied several synchronization policies between a single vendor and multiple buyers. Jha and Shanker [28] presented an integrated production-inventory model formulated to minimize the joint total expected cost.

Additionally, a substantial number of researches devoted to the study on vendor-managed inventory (VMI). Some representative researches on a single-vendor multi-buyer vendor-managed inventory summarize as follows. Lu [31] presented a heuristic approach to solve a single-vendor multi-buyer integrated inventory problem involving equal shipment sizes. Zhanga et al. [45] developed an integrated VMI model for a single vendor and multiple buyers. Darwish and Odah [14] considered a VMI model for a single vendor multi-retailer supply chain problem in which the vendor incurs a penalty for items in excess of definite bounds. Zavanella and Zanoni [44] extended the application of a consignment policy in which the retailer pays the vendor for each item he sells to a one-vendor multi-buyer production-distribution system. Yu et al. [43] devised a single-manufacturer (vendor) multi-retailer (buyer) generic model, based on the consignment policy, under stochastic customer demand. Hariga et al. [19] considered a centralized supply chain composed of a single vendor serving multiple buyers and operating under a consignment stock arrangement. Srinivas and Rao [40] established four consignment stock inventory models for a single-vendor-multi-buyer supply chain.

Battini et al. [5] dealt with a multi-echelon inventory system in which one vendor supplies an item to multiple buyers, and developed a consignment stock inventory model in which many clients can establish a consignment stock policy with the same vendor. Ben-Daya et al. [7] modeled a consignment and vendor-managed inventory policy for a single-vendor, multi-buyer supply chain with known demand and studied three vendor-buyer partnerships. Chen et al. [13] dealt with the problem of coordinating a vertically separated distribution system under vendor-managed inventory and consignment arrangements with one wholesaler and multiple non-identical retailers. Adida and Ratisoontorn [1] investigated how competition among retailers influences supply chain decisions and profits under different consignment arrangements. Sarker [37] studied consignment stocking policy models for supply chain systems and comprehensively surveyed these models and performed associated critical analyses. Hariga and Al-Ahmari [18] designed integrated retail shelf space allocation and inventory models for a supply chain operating under a vendor-managed inventory and consignment stock agreement. Mateen et al. [33] discussed the interaction involving replenishment cycle between a vendor and multiple retailers in a VMI system under stochastic demand. Glock and Kim [16] studied a single-vendor-multi-retailer supply chain and considered the case where the vendor merges with one of its retailers, and indicated that the type of competition is of crucial importance for the structure of the supply chain after merging. However, as we realized, these foregoing studies did not simultaneously address the inventory and allocation decisions problem for vendors.

Another important issue is the variability of market demand. Historically it was commonly accepted that random demand frequently occurs in competitive commodities like perishable items. It is especially true for perishable items because of a short life cycle. Recent comparable studies on the newsvendor-type inventory model were interested in the probabilistic random demand. Furthermore, most probabilistic demand-related studies adopt independent normal demand for individual time periods. As we realized, Bagchi and Hayya [2], Bagchi et al. [3], Silver et al. [39], Mantrala and Raman [32], Tang et al. [41], McCardle et al. [34], Chen and Chen [10, 11], Desmet et al. [15], and Jha and Shanker [28] assumed normally distributed demand in their analytical models. Using a normal distribution to model demand on a given product appears to be convincing because market demand is regularly an aggregate of numerous individual demand and thus will most likely approximate a normal demand.

Nevertheless, it is questionable and unsuitable that normal distribution is taken as a proxy for demand distribution as a result of the impossibility of negative demand. Bartezzaghi et al. [4] argued that a probability distribution should be sought to imitate demand distribution if its field is defined only for non-negative values. Among others, the lognormal distribution is selected and considered as a more acceptable and valid alternative to the normal distribution. The reason is that a variable with a lognormal distribution only takes a value between zero and infinity and results in a normal distribution following the logarithmic operation, which is relatively easy to manipulate mathematically. Consequently, assuming that market demand for perishable items follows a lognormal distribution should be theoretically justifiable and sustainable. Benavides et al. [6], Huang et al. [25], and Huang [23, 24] also supported the lognormal distribution, and used it to analyze and address the problems of demand forecasting. Presumably more studies will make use of the lognormal distribution to model the demand variable in the future.

In sum, this study considers the situation of a vendor responsible for producing and delivering a quantity that matches expected aggregate demand from multiple retailers during an upcoming selling period for a given perishable item and allocates this perishable inventory among retailers. For developing the optimal inventory-allocation decision for vendor under above circumstance, this study seeks to extend the newsvendor model to create the integrated inventory-allocation decision model for a given perishable item associated with a number of independent lognormal demands from retailers. Notably, besides lognormal demand distribution, the proposed model also incorporates Ito process to capture demand manner and a comprehensive integrated approach for deducing the aggregate demand. Finally, an effective and practical analytic method is developed to assist vendors of perishable items determine the inventory quantity and allocation decision that can maximize the expected profits. Through numerical examples, this study demonstrates that the optimal inventory-allocation solution can be straightforwardly obtained using the proposed inventory-allocation integrated model. Furthermore, a finding via sensitivity analysis is that the expected profits increase with decreased volatility of aggregate demand that can be realized by means of encouraging retail competition to reduce demand correlation between retailers.

2. Analytical Model Development

The purpose of this section is to develop a more plausible, effective and practicable vendor-managed inventory-allocation integrated model, which can maximize vendor' profit, for given perishable item with stochastic demand in a single-vendor multi-retailer supply chain.

2.1. Modeling demand forecast

In contrast with the previous studies, the distinguishable characteristics in modeling demand forecast is as follows: (1) market demand for considered perishable items is assumed to comply with the lognormal distribution. (2) Ito process, which is usually used to model the behavior of financial assets price, is applied to suitably capture the demand shift by using the corresponding continuous-time differential equation. This study is confident that these two characteristics are better off in imitating and modeling the variability of real-life demand for a given perishable item. The Ito process is a stochastic process that possesses the Markov property and involves with a permanent component of regular trend and a temporary component of random diffusion. Let D_t represent demand quantity during period t for a given perishable item. The Ito process for random variable D_t can then be represented algebraically as follows:

$$dD_t = \mu(D_t, t)dt + \sigma(D_t, t)dz_t. \tag{2.1}$$

In the stochastic diffusion equation of Eq. (2.1), $\mu(D_t, t)$ represents the regular trend component, while $\sigma(D_t, t)$ represents the random diffusion component. Additionally, variable z_t is assumed to satisfy the standard Wiener process (or say, Brownian motion), and its increment $dz_t = z_t - z_{t-1} = \varepsilon_t\sqrt{\tau}$, where ε_t is a standard normal variable; that is, $\varepsilon_t \sim \phi(0, 1)$ and $d\tau$ represents a given small time interval, accordingly implying that dz_t is a normal variable; namely $dz_t \sim \phi(0, \tau)$.

A special form $\mu(D_t, t) = \mu D_t$ and $\sigma(D_t, t) = \sigma D_t$ can be obtained when the diffusion component is roughly stationary (i.e., independent of time) that is expected to hold for the demand process of a typical perishable item over a relatively finite selling horizon. Where, parameters μ and σ represent the expected rate of demand growth and the standard deviation of the rate of demand growth, respectively, and both are constant for all time periods. Eq. (2.1), thus, can be reformulated as follows:

$$dD_t = \mu D_t dt + \sigma D_t dz_t. \tag{2.2}$$

Assume that D_t is a lognormally distributed variable and let $f = \ln D_t$. Taylor expansion is applied to f with truncating all but first two terms and then incorporated to Eq. (2.2) to yield the following discrete-time model:

$$D_t = D_0 \exp\Big[\Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma\varepsilon\sqrt{t}\Big], \tag{2.3}$$

where

$D_0 =$ demand quantity during previous selling period,

$t =$ length of time period,

$\mu =$ expected annual rate of demand growth,

$\sigma =$ standard deviation of rate of demand growth,

$\varepsilon =$ a standard normal random variable; that is, $\varepsilon \sim \phi(0, 1)$.

Eq. (2.3) explicitly indicates that the expected value and variance of $\ln D_t$ are $E[\ln D_t] = \ln D_0 + (\mu - \sigma^2/2)$ and $\text{Var}\,[\ln D_t] = \sigma^2 t$, respectively.

2.2. Optimal inventory-allocation integrated model

The symbols used in the analytical model are defined as follows.

$m =$ number of retailers,

$T =$ length of selling period for a given perishable item (unit: years),

$c =$ unit production/purchase cost (wholesale price) for a given perishable item,

$p =$ unit selling price (retail price) for a given perishable item, which is equal for all retailers,

$v =$ unit commission paid by the vendor to retailers, which is equal for all retailers,

$h =$ unit holding cost for a given perishable item per period,

$s =$ unit salvage value for a given perishable item ($s < c < p$),

$r =$ unit shortage cost for a given perishable item ($r \geq p - c$, shortage cost would equal to opportunity cost (selling profit) plus potential goodwill loss),

$D_i =$ demand quantity from retailer i during the upcoming selling period for a given perishable item,

$D_{0,i} =$ actual demand from retailer i during the previous selling period for a given perishable item,

$b_i =$ pre-unit adjustment cost from retailer i for a given perishable item,

$\mu_i =$ expected annual demand growth rate from retailer i for a given perishable item,

$\sigma_i =$ standard deviation of demand growth rate from retailer i for a given perishable item, implying demand volatility for retailer i,

$\sigma_{ij} =$ covariance of demand between retailers i and j for a given perishable item, implying demand correlation between retailers i, and j.

This study first assumes that the vendor has sufficient capacity to satisfy all retailer demand. Also, quantity adjustment whereby retailers with excessive inventory transfer part inventory to those retailers who turn up a shortage is permitted and promoted. After doing so, the inventory quantity of perishable items in the supply chain can be utilized more efficiently to cut down unsold units and goodwill damage. As mentioned above, there are two decisions having to be adequately made for vendor in order to boost profitability. On the one hand, the inventory quantity approximates as closely as possible the aggregate demand during the upcoming selling period. On the other hand, inventory allocation among retailers match as accurately as possible to the demand of each retailer so as to lessen adjustment cost.

Let Q_i denote inventory-allocation quantity for retailer i during the upcoming selling period, which is the decision variable in this study. In this study, vendor profit is set equaling to subtract adjustment cost from selling gross profit. Expressed formally, the net profit function during the upcoming selling period for vendor depends on the aggregate demand and adjustment cost, and can be formulated as follows:

$$R = \begin{cases} (p-c-v-h)\sum_{i=1}^{m} D_i - (c+h-s)\Big(\sum_{i=1}^{m} Q_i - \sum_{i=1}^{m} D_i\Big) & \\ -\Big[\sum_{i=1}^{m} b_i \operatorname{Max}(Q_i - D_i, 0)\Big]; & 0 \le \sum_{i=1}^{m} D_i \le \sum_{i=1}^{m} Q_i \\ (p-c-v-h)\sum_{i=1}^{m} Q_i - r\Big(\sum_{i=1}^{m} D_i - \sum_{i=1}^{m} Q_i\Big) & \\ -\Big[\sum_{i=1}^{m} b_i \operatorname{Max}(Q_i - D_i, 0)\Big]; & \sum_{i=1}^{m} D_i > \sum_{i=1}^{m} Q_i \end{cases}$$

or, say

$$R = \begin{cases} (p-s-v)D_S - (c+h-s)Q_S - \Big[\sum_{i=1}^{m} b_i \operatorname{Max}(Q_i - D_i, 0)\Big]; \; 0 \le D_S \le Q_S \\ (p+r-c-v-h)Q_S - rD_S - \Big[\sum_{i=1}^{m} b_i \operatorname{Max}(Q_i - D_i, 0)\Big]; \quad D_S > Q_S \end{cases}. \tag{2.4}$$

Where the aggregate demand $D_S = \sum_{i=1}^{m} D_i$ and aggregated inventory quantity $Q_S = \sum_{i=1}^{m} Q_i$. It should be highlighted that the following novel integrated approach for the aggregate demand is another noticeable device and is confident of being contributive and referable to the future researches. Because the sum of a set of lognormal variables is not a lognormal, the aggregate demand D_S has to be equivalently transformed with the purpose of solving the geometric mean, which will be explained later, as follows.

$$\begin{aligned} D_S &= \sum_{i=1}^{m} D_i = \sum_{i=1}^{m} \Big(E[D_i] \times \frac{D_i}{E[D_i]}\Big) \\ &= \sum_{i=1}^{m} \left(\frac{E[D_i]}{\sum_{j=1}^{m}(E[D_j])} \times \frac{D_i}{E[D_i]}\right) \times \sum_{j=1}^{m}(E[D_j]) \end{aligned}$$

$$= \sum_{j=1}^{m} E[D_j] \times \sum_{i=1}^{m} (w_i \times D_i^*) \tag{2.5}$$

where

$$w_i = \frac{E[D_i]}{\sum\limits_{j=1}^{m} E[D_j]} \quad \text{and} \quad \sum_{i=1}^{m} w_i = 1,$$

$$D_i^* = \frac{D_i}{E[D_i]},$$

$$E[D_i] = D_{0,i} \times e^{\mu_i T}.$$

The arithmetic average of a set of lognormally distributed variables is not lognormal too whereas the geometric average is lognormal. Consequently, the arithmetic average must be replaced by the geometric average as so to fit for Eq. (2.3) through the following approximate transformation.

$$\sum_{i=1}^{m} (w_i \times D_i^*) \cong \prod_{i=1}^{m} (D_i^*)^{w_i} - E\Big[\prod_{i=1}^{m} (D_i^*)^{w_i}\Big] + E\Big[\sum_{i=1}^{m} (w_i \times D_i^*)\Big], \tag{2.6}$$

where,

$$\prod_{i=1}^{m} (D_i^*)^{w_i} = \prod_{i=1}^{m} \Big(\frac{D_i}{E[D_i]}\Big)^{w_i} = \prod_{i=1}^{m} \Big(\frac{D_i}{D_{0,i}}\Big)^{w_i} \times \prod_{i=1}^{m} (e^{-\mu_i T})^{w_i}$$

$$= \prod_{i=1}^{m} \Big(\frac{D_i}{D_{0,i}}\Big)^{w_i} \times \Big(e^{\sum\limits_{i=1}^{m}(-w_i \mu_i T)}\Big), \tag{2.7}$$

$$E\Big[\sum_{i=1}^{m} (w_i \times D_i^*)\Big] = \sum_{i=1}^{m} (w_i \times E[D_i^*]) = \sum_{i=1}^{m} \Big(w_i \times E\Big[\frac{D_i}{E[D_i]}\Big]\Big) = \sum_{i=1}^{m} w_i = 1. \tag{2.8}$$

Based on (2.7), it can be straightforwardly worked out that

$$\ln\Big(\prod_{i=1}^{m} (D_i^*)^{w_i}\Big) = \sum_{i=1}^{m} w_i \times \ln\Big(\frac{D_i}{D_{0,i}}\Big) - \sum_{i=1}^{m} (w_i \mu_i T). \tag{2.9}$$

Moreover, because $\ln(D_i/D_{0,i})$; $i = 1, 2, \ldots, m$ within Eq. (2.9) are all normal distributions; that is, conforming to $\phi((\mu_i - \sigma_i^2/2)T, \sigma_i^2 T)$, it follows that $w_i \times \ln(D_i/D_{0,i})$ conform to normal distributions of $\phi(w_i \times (\mu_i - \sigma_i^2/2)T, w_i^2 \sigma_i^2 T)$; $i = 1, 2, \ldots, m$ as well. For the sake of simplicity, let $X = \prod\limits_{i=1}^{n} (D_i^*)^{w_i}$, and then the following two expressions can be derived out on the basis of the definition of mean and variance.

$$E\left[\ln\Big(\prod_{i=1}^{m} (D_i^*)^{w_i}\Big)\right] = E[\ln X] = \sum_{i=1}^{m} [w_i(\mu_i - \sigma_o i^2/2)T] - \sum_{i=1}^{m} (w_i \mu_i T)$$

$$= \sum_{i=1}^{m} (-w_i \sigma_i^2/2)T = \mu_X T, \tag{2.10}$$

$$\mathrm{Var}\left[\ln\Big(\prod_{i=1}^{m}(D_i^*)^{w_i}\Big)\right] = \mathrm{Var}\,[\ln X] = \left(\sum_{i=1}^{m}[w_i^2\times\sigma_i^2] + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}[w_i\times w_j\times\sigma_{ij}]\right)T$$
$$= \sigma_X^2 T, \tag{2.11}$$

or following a shifting operation, say

$$\mu_X = \sum_{i=1}^{m}(-w_i\sigma_i^2/2),$$
$$\sigma_X == \left(\sum_{i=1}^{m}[w_i^2\times\sigma_i^2] + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}[w_i\times w_j\times\sigma_{ij}]\right)^{1/2}.$$

Thus it turns out that

$$E\Big[\prod_{i=1}^{m}(D_i^*)^{w_i}\Big] = E[X] = e^{\mu_X T+\sigma_X^2 T/2}. \tag{2.12}$$

Eq. (2.6) can be rewritten according to Eq. (2.12) as follows.

$$\sum_{i=1}^{m}(w_i\times D_i^*) \cong \prod_{i=1}^{m}(D_i^*)^{w_i} - E\Big[\prod_{i=1}^{m}(D_i^*)^{w_i}\Big] + E\Big[\sum_{i=1}^{m}(w_i\times D_i^*)\Big] = X - A + 1, \tag{2.13}$$

where,

$$A == E[X] = e^{\mu_X T+\sigma_X^2 T/2}.$$

Letting $B = \sum_{i=1}^{m}(E[D_i])$, then in accordance with Eq. (2.5) the aggregate demand can be re-expressed as follows

$$D_S = \sum_{i=1}^{m}(E[D_i])\times\sum_{i=1}^{m}(w_i\times D_i^*) = B\times(X - A + 1). \tag{2.14}$$

Following the above result substitutes for D_S in Eq. (2.4), the net profit function of vendor thus can be reformulated as follows.

$$R = \begin{cases} (p-s-v)B\times X - (p-s-v)B(A-1) - (c+h-s)Q_S \\ -\Big[\sum_{i=1}^{m} b_i\,\mathrm{Max}(Q_i - D_i, 0)\Big]; & 0\le X\le Q_S/B + A - 1 \\ [(p+r-c-v-h)Q_S + rB(A-1)] - rB\times X \\ -\Big[\sum_{i=1}^{m} b_i\,\mathrm{Max}(Q_i - D_i, 0)\Big]; & X > Q_S/B + A - 1. \end{cases} \tag{2.15}$$

Accordingly, vendor expected profit is then deduced from Eq. (2.15) and can be expressed according to the definition of expected value as follows:

$$E[R] = (p-s-v)B\int_0^{Q_S/B+A-1} Xf(X)dX$$

$$-[(p-s-v)B(A-1)+(c+h-s)Q_S]\int_0^{Q_S/B+A-1} f(X)dX$$
$$+[(p+r-c-v-h)Q_S+rB(A-1)]\int_{Q_S/B+A-1}^{\infty} f(X)dX$$
$$-rB\int_{Q_S/B+A-1}^{\infty} Xf(X)dX$$
$$-\Big\{\sum_{i=1}^{m} b_i\Big[\Big(Q_i\int_0^{Q_1} f(D_i)dD_i-\int_0^{Q_1} D_i f(D_i)dD_i\Big)$$
$$+\Big(\int_{Q_1}^{\infty} D_i f(D_i)dD_i-Q_i\int_{Q_1}^{\infty} f(D_i)dD_i\Big)\Big]\Big\}. \tag{2.16}$$

As stated previously, the demand (D_i and D_i^*) from each retailer during a certain selling period for a given perishable item is assumed to follow a lognormal distribution, and thus the probability distribution for the geometric average (X) of demand (D_i^*) is also lognormal distribution. Thus, the probability density function of variables D_i and X can be respectively expressed as follows:

$$f(D_i)=\frac{1}{D_i}\frac{1}{\sigma_{D_i}\sqrt{T}\sqrt{2\pi}}e^{\dfrac{-(\ln D_i-E[\ln D_i])^2}{2\sigma_{D_i}^2 T}},$$
$$f(X)=\frac{1}{X}\frac{1}{\sigma_X\sqrt{T}\sqrt{2\pi}}e^{\dfrac{-(\ln X-E[\ln X])^2}{2\sigma_X^2 T}}.$$

Theorem. *Expected profit of vendor is worked out as follows:*

$$E[R]=(p+r-s-v)\Big\{\Big[Q_S+AB-B]N(d_{01})-ABN(d_{02})\}+(p-s-v)B-(c+h-s)Q_S$$
$$-\sum_{i=1}^{m} b_i\Big[2\Big(D_{0,i}e^{\mu_{D_i}T}N(d_{i2})-Q_iN(d_{i1})\Big)+Q_i-D_{0,i}e^{\mu_{D_i}T}\Big], \tag{2.17}$$

where,

$$d_{01}=\frac{\ln[B/(Q_S+AB-B)]+\mu_X T}{\sigma_X\sqrt{T}},$$
$$d_{02}=\frac{\ln[B/(Q_S+AB-B)]+(\mu_X+\sigma_X^2)T}{\sigma_X\sqrt{T}},$$
$$d_{i1}=\frac{\ln[D_{0,i}/Q_i]+(\mu_{D_i}-\sigma_{D_i}^2/2)T}{\sigma_{D_i}\sqrt{T}},$$
$$d_{i2}=\frac{\ln[D_{0,i}/Q_i]+(\mu_{D_i}+\sigma_{D_i}^2/2)T}{\sigma_{D_i}\sqrt{T}}.$$

Where, function $N(x)$ is the cumulative distribution function for a standardized normal variable. In other words, it is the probability that a variable with a standard normal distribution, $\phi[0,1]$, will be less than x.

Proof. The deduction is relegated to Appendix.

Taking the first derivative of $E[R]$ with respect to order quantity Q_i; $i = 1, 2, \ldots, m$ reveals that

$$\frac{\partial E[R]}{\partial Q_i} = (p + r - s - v)\Big[N(d_{01}) - \frac{n(d_{01})}{\sigma_X \sqrt{T}} + AB\frac{n(d_{02})}{\sigma_X \sqrt{T}(Q_S + AB - B)}\Big] - (c + h - s)$$
$$-b_i\Big[2\Big(- D_{0,i}e^{\mu_{D_i}T}\frac{n(d_{i2})}{\sigma_{D_i}\sqrt{T}Q_i} - N(d_{i1}) + \frac{n(d_{i1})}{\sigma_{D_i}\sqrt{T}}\Big) + 1\Big];\ i = 1, 2, \ldots, m. \quad (2.18)$$

Where $n(d_{01}) = \frac{1}{\sqrt{2\pi}}e^{-d_{01}^2/2}$, $n(d_{02}) = \frac{1}{\sqrt{2\pi}}e^{-d_{02}^2/2}$, $n(d_{i1}) = \frac{1}{\sqrt{2\pi}}e^{-d_{i1}^2/2}$, and $n(d_{i2}) = \frac{1}{\sqrt{2\pi}}e^{-d_{i2}^2/2}$. Moreover, $n(d_{02})$ can be substituted for $n(d_{i2})$ via the following transformation

$$\begin{aligned} n(d_{01}) &= \frac{1}{\sqrt{2\pi}}e^{-d_{01}^2/2} = \frac{1}{\sqrt{2\pi}}e^{-(d_{02} - \sigma_X \sqrt{T})^2/2} \\ &= \frac{1}{\sqrt{2\pi}}\Big(e^{-d_{02}^2/2 + \sigma_X \sqrt{T} \times d_{02} - \sigma_X^2 T/2}\Big) \\ &= \frac{1}{\sqrt{2\pi}}\Big(e^{-d_{02}^2/2 + \ln[B/(Q_S + AB - B)] + (\mu_X + \sigma_X^2/2)T}\Big) \\ &= AB \times \frac{n(d_{02})}{Q_S + AB - B}. \end{aligned}$$

Likewise, through similar logic $n(d_{i1})$ can be substituted with $n(d_{i2})$ via the following transformation

$$n(d_{i1}) = D_{0i}e^{\mu_{D_i}T} \times \frac{n(d_{i2})}{Q_i}.$$

Consequently, it turns out that

$$\frac{\partial E[R]}{\partial Q_i} = (p + r - s - v)N(d_{01}) - (c + h - s) - b_i[1 - 2N(d_{i1})];\ i = 1, 2, \ldots, m. \quad (2.19)$$

The maximal value of $E[R^*]$ occurs at Q_i^*, satisfying jointly $\partial E[R]/\partial Q_i = 0$; $i = 1, 2, \ldots, m$. Therefore, the optimal inventory-allocation decision Q_i^*; $i = 1, 2, \ldots, m$ are determined first through simultaneously resolving $\partial E[R]/\partial Q_i = 0$; $i = 1, 2, \ldots, m$. Although a closed-form expression cannot be obtained, some available numerical software can facilitate to find out the solution. Subsequently, the maximal expected profit $E[R^*]$ can be estimated using Eq. (2.17), immediately after determining Q_i^*; $i = 1, 2, \ldots, m$.

Finally, this study takes a supply chain system composed of two retailers i and j as an illustration to prove the concavity of proposed analytic model. Since

$$U = \frac{\partial^2 E[R]}{\partial Q_i^2} = -\Big[(p + r - s - v)\frac{n(d_{01})}{\sigma_X \sqrt{T}(Q_S + AB - B)} + 2b_i\frac{n(d_{i1})}{\sigma_{D_i}\sqrt{T}Q_i}\Big] < 0,$$
$$V = \frac{\partial^2 E[R]}{\partial Q_i \partial Q_j} = -\Big[(p + r - s - v)\frac{n(d_{01})}{\sigma_X \sqrt{T}(Q_S + AB - B)}\Big],$$

$$W = \frac{\partial^2 E[R]}{\partial Q_j^2} = -\Big[(p+r-s-v)\frac{n(d_{01})}{\sigma_X\sqrt{T}(Q_S+AB-B)} + 2b_j\frac{n(d_{j1})}{\sigma_{D_j}\sqrt{T}Q_i}\Big],$$

$$Y = V^2 - UW = -\Big[\frac{(p+r-s-v)n(d_{01})}{\sigma_X\sqrt{T}(Q_S+AB-B)} \times 2\Big(\frac{b_i n(d_{i1})}{\sigma_{D_i}\sqrt{T}Q_i} + \frac{b_j n(d_{j1})}{\sigma_{D_j}\sqrt{T}Q_j}\Big) + 4\frac{b_i b_j n(d_{i1})n(d_{j1})}{\sigma_{D_i}\sigma_{D_j}TQ_iQ_j}\Big] < 0,$$

the existence of maximal value can thus be verified.

2.3. Model parameters estimation

The expected growth rate of demand μ_i, standard deviation of growth rate σ_i, and covariance σ_{ij} can be determined from the sample estimates $\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\sigma}_{ij}$, respectively, based on historical retailer demand data. Assuming two samples of demand data for retailers i and j for past N time periods, namely, $D_{i1}, D_{i2}, \ldots, D_{iN}$ and $D_{j1}, D_{j2}, \ldots, D_{jN}$, each with length $\Delta\tau$, for a given perishable item, the logarithmic growth rate of demand r_t for the demand time series during period t is as follows:

$$r_{kt} = \ln\Big(\frac{D_{kt}}{D_{kt-1}}\Big); \quad t = 2, 3, \ldots, N, \quad k = i, j. \tag{2.20}$$

Additionally, for retailer k ($k = i, j$) and during period t, the logarithmic growth rate of demands r_{kt}, $t = 2, 3, \ldots, N$ all share an independent and identical normal distribution with mean $\bar{r}_k$ and standard deviation s_k.

Accordingly, for retailer k; $k = i, j$, and the three estimates $\hat{\mu}_k$, $\hat{\sigma}_k$ and $\hat{\sigma}_{ij}$ can be calculated as follows:

$$\hat{\mu}_k = \frac{\bar{r}_k}{\Delta\tau} + \frac{s_k^2}{2\Delta\tau}, \quad \text{and } \hat{\sigma}_k = \frac{s_k}{\sqrt{\Delta\tau}}, \tag{2.21}$$

$$\bar{r}_k = \frac{\sum_{t=2}^{N} r_{kt}}{N-1}, \quad \text{and } s_k = \sqrt{\frac{\sum_{t=2}^{N}(r_{kt}-\bar{r}_k)^2}{N-2}}, \tag{2.22}$$

$$\hat{\sigma}_{ij} = \frac{\sum_{t=2}^{N}(r_{it}-\bar{r}_i)(r_{jt}-\bar{r}_j)}{N-2}. \tag{2.23}$$

3. Numerical Experiment

This section presents a numerical example involving a supply chain that consists of one vendor and five retailers, and demonstrates that the proposed analytical model can optimize inventory quantity and inventory-allocation decisions to maximize the expected profit for vendor. Meanwhile, the model parameters required in the numerical example are arranged for a plausible perishable item, yielding a corresponding value set comprising $(T, p, c, v, h, s, r) = (0.5, \$100, \$60, \$15, \$2, \$10, \$150)$. Additionally, the following demand quantities during the previous selling period, expected annual demand growth rates, adjustment cost per unit, and variance and covariance of demand growth rates for those five retailers are hypothetically acquired from historical demand data based on Section 2.3 and used in this example:

$$D_0 = \begin{bmatrix} 10,000 \\ 15,000 \\ 30,000 \\ 8,000 \\ 50,000 \end{bmatrix}, \mu = \begin{bmatrix} 0.15 \\ 0.2 \\ 0.5 \\ -0.1 \\ 0.3 \end{bmatrix}, b = \begin{bmatrix} \$2 \\ \$5 \\ \$1 \\ \$8 \\ \$3 \end{bmatrix}, \sigma = \begin{bmatrix} 0.0400 & 0.0420 & -0.0100 & 0.0120 & -0.0300 \\ 0.0420 & 0.1225 & 0.0263 & -0.0735 & 0.0750 \\ -0.0100 & 0.0263 & 0.0625 & 0.0750 & 0.0188 \\ 0.0120 & 0.0735 & -0.0750 & 0.3600 & 0.1350 \\ -0.0300 & 0.0750 & 0.0188 & 0.1350 & 0.2500 \end{bmatrix}.$$

By using the numerical program offered by MATLAB software, the optimal inventory-allocation decision and relative percentage for every retailer i, $i = 1, 2, \ldots, 5$ for the above parameter settings using the proposed analytical method and numerical solution procedure are readily solved as

$$Q^* = \begin{bmatrix} 11,065 \\ 16,486 \\ 41,647 \\ 7,144 \\ 57,942 \end{bmatrix}, \text{ and } \gamma_{Q^*} \begin{bmatrix} 0.0824 \\ 0.1228 \\ 0.3101 \\ 0.0532 \\ 0.4315 \end{bmatrix}.$$

Meanwhile, the optimal aggregated inventory quantity and expected profit for vendor are calculated as $Q_S^* = 134,283$ and $E[R^*] = \$1,636,950$, respectively.

To examine concavity this study presents two scenarios to compare the expected profits. The first scenario designates a given range of aggregated inventory quantities, each which is allocated among five retailers based on the optimal relative percentage $\gamma_{Q_i^*}$. The second scenario involves the aggregated inventory quantity remaining the same as Q_S^* for all cases and inventory-allocation decisions for these five retailers are sequentially adjusted in pairs, where each pair comprises one variable that increases by a certain percentage and another that decreases by an identical percentage relative to the identified optimal relative percentage $\gamma_{Q_i^*}$.

While Table 1 lists the corresponding expected profits for the first scenario, in which aggregated inventory quantities range from 90,000 to 180,000 in increments of 2,500, and Table 2 lists numerical results for the second scenario, in which the inventory-allocation quantity of each retailer is adjusted in turn by percentages of 0.1%, 0.5% and 1%. Based on the results listed in Table 1, Figure 1 illustrates the variability of expected profit, clearly demonstrating that it first increases and then decreases with rising inventory quantities, ultimately reaching (as expected) a maximum of \$1,636,950 for a inventory quantity of 134,283, thus validating the solution and confirming the concavity.

Besides, Figure 1 illustrates that expected profits for smaller inventory quantities tend to be lower than those for larger inventory quantities owing to the shortage cost considerably exceeding the salvage value. On the other hand, the examination of expected profit shown in Table 2 also clearly reveals that, as expected, none of the fine-tuned

Table 1: Comparison of expected profits for a given range of inventory quantities.

Q_S	Q_1	Q_2	Q_3	Q_4	Q_5	$E[R]$
90,000	7,416	11,049	27,913	4,788	38,834	-172,583
92,500	7,622	11,356	28,688	4,921	39,913	2,403
95,000	7,828	11,663	29,463	5,054	40,992	172,943
97,500	8,034	11,970	30,239	5,187	42,070	338,140
100,000	8,240	12,277	31,014	5,320	43,149	497,055
102,500	8,446	12,584	31,789	5,453	44,228	648,724
105,000	8,652	12,891	32,565	5,586	45,306	792,194
107,500	8,858	13,198	33,340	5,719	46,385	926,545
110,000	9,064	13,505	34,115	5,852	47,464	1,050,919
112,500	9,270	13,812	34,891	5,985	48,543	1,164,545
115,000	9,476	14,119	35,666	6,118	49,621	1,266,757
117,500	9,682	14,426	36,441	6,251	50,700	1,357,011
120,000	9,888	14,732	37,217	6,384	51,779	1,434,896
122,500	10,094	15,039	37,992	6,517	52,857	1,500,138
125,000	10,300	15,346	38,767	6,650	53,936	1,552,602
127,500	10,506	15,653	39,543	6,783	55,015	1,592,286
130,000	10,712	15,960	40,318	6,916	56,094	1,619,314
132,500	10,918	16,267	41,094	7,049	57,172	1,633,927
134,283	**11,065**	**16,486**	**41,647**	**7,144**	**57,942**	**1,636,950**
135,000	11,124	16,574	41,869	7,182	58,251	1,636,468
137,500	11,330	16,881	42,644	7,315	59,330	1,627,364
140,000	11,536	17,188	43,420	7,448	60,409	1,607,119
142,500	11,742	17,495	44,195	7,581	61,487	1,576,289
145,000	11,948	17,802	44,970	7,714	62,566	1,535,476
147,500	12,154	18,109	45,746	7,847	63,645	1,485,305
150,000	12,360	18,416	46,521	7,980	64,723	1,426,418
152,500	12,566	18,723	47,296	8,113	65,802	1,359,461
155,000	12,772	19,029	48,072	8,246	66,881	1,285,071
157,500	12,978	19,336	48,847	8,379	67,960	1,203,871
137,500	11,330	16,881	42,644	7,315	59,330	1,627,364
160,000	13,184	19,643	49,622	8,512	69,038	1,116,463
162,500	13,390	19,950	50,398	8,645	70,117	1,023,420
165,000	13,596	20,257	51,173	8,778	71,196	925,285
167,500	13,802	20,564	51,948	8,911	72,275	822,569
170,000	14,008	20,871	52,724	9,044	73,353	715,745
172,500	14,214	21,178	53,499	9,177	74,432	605,253
175,000	14,420	21,485	54,274	9,310	75,511	491,495
177,500	14,626	21,792	55,050	9,443	76,589	374,840
180,000	14,832	22,099	55,825	9,576	77,668	255,621

inventory-allocation decisions are superior to the identified optimal inventory-allocation relative percentage $\gamma_{Q_i^*}$. Two additional comparisons are performed in Table 2, provided $\gamma_{Q_i^*}$ is based respectively on $D_{0,i}$ and $D_{0,i}e^{\mu_i T}$, and their performances are also inferior. In conclusion, the concavity of the proposed analytical model can be identified by reason of evidences from two experimental scenarios.

Table 2: Comparison of expected profits for various inventory-allocation decisions under three fine-tuning rates.

Q_S	Q_1	Q_2	Q_3	Q_4	Q_5	$E[R]$
Optimal	**11,065**	**16,486**	**41,647**	**7,144**	**57,942**	**1,636,950**
0.1%	11,199	16,352	41,647	7,144	57,942	1,636,933
	11,199	16,486	41,512	7,144	57,942	1,636,941
	11,199	16,486	41,647	7,010	57,942	1,636,922
	11,199	16,486	41,647	7,144	57,807	1,636,940
	10,930	16,620	41,647	7,144	57,942	1,636,933
	11,065	16,620	41,512	7,144	57,942	1,636,941
	11,065	16,620	41,647	7,010	57,942	1,636,922
	11,065	16,620	41,647	7,144	57,807	1,636,941
	10,930	16,486	41,781	7,144	57,942	1,636,940
	11,065	16,352	41,781	7,144	57,942	1,636,941
	11,065	16,486	41,781	7,010	57,942	1,636,930
	11,065	16,486	41,781	7,144	57,807	1,636,948
	10,930	16,486	41,647	7,278	57,942	1,636,922
	11,065	16,352	41,647	7,278	57,942	1,636,923
	11,065	16,486	41,512	7,278	57,942	1,636,931
	11,065	16,486	41,647	7,278	57,807	1,636,930
	10,930	16,486	41,647	7,144	58,076	1,636,940
	11,065	16,352	41,647	7,144	58,076	1,636,940
	11,065	16,486	41,512	7,144	58,076	1,636,948
	11,065	16,486	41,647	7,010	58,076	1,636,930
0.5%	11,736	15,815	41,647	7,144	57,942	1,636,520
	11,736	16,486	40,975	7,144	57,942	1,636,721
	11,736	16,486	41,647	6,473	57,942	1,636,253
	11,736	16,486	41,647	7,144	57,270	1,636,717
	10,393	17,157	41,647	7,144	57,942	1,636,504
	11,065	17,157	40,975	7,144	57,942	1,636,714
	11,065	17,157	41,647	6,473	57,942	1,636,246
	11,065	17,157	41,647	7,144	57,270	1,636,709
	10,393	16,486	42,318	7,144	57,942	1,636,698
	11,065	15,815	42,318	7,144	57,942	1,636,706
	11,065	16,486	42,318	6,473	57,942	1,636,440
	11,065	16,486	42,318	7,144	57,270	1,636,903
	10,393	16,486	41,647	7,816	57,942	1,636,263
	11,065	15,815	41,647	7,816	57,942	1,636,271
	11,065	16,486	40,975	7,816	57,942	1,636,473
	11,065	16,486	41,647	7,816	57,270	1,636,468
	10,393	16,486	41,647	7,144	58,613	1,636,692
	11,065	15,815	41,647	7,144	58,613	1,636,701
	11,065	16,486	40,975	7,144	58,613	1,636,903
	11,065	16,486	41,647	6,473	58,613	1,636,435
1%	12,407	15,143	41,647	7,144	57,942	1,635,287
	12,407	16,486	40,304	7,144	57,942	1,636,102
	12,407	16,486	41,647	5,801	57,942	1,634,183
	12,407	16,486	41,647	7,144	56,599	1,636,085
	9,722	17,829	41,647	7,144	57,942	1,635,175
	11,065	17,829	40,304	7,144	57,942	1,636,024
	11,065	17,829	41,647	5,801	57,942	1,634,105
	11,065	17,829	41,647	7,144	56,599	1,636,007
	9,722	16,486	42,989	7,144	57,942	1,635,932
	11,065	15,143	42,989	7,144	57,942	1,635,966
	11,065	16,486	42,989	5,801	57,942	1,634,861
	11,065	16,486	42,989	7,144	56,599	1,636,764
	9,722	16,486	41,647	8,487	57,942	1,634,267
	11,065	15,143	41,647	8,487	57,942	1,634,300
	11,065	16,486	40,304	8,487	57,942	1,635,115
	11,065	16,486	41,647	8,487	56,599	1,635,099
	9,722	16,486	41,647	7,144	59,285	1,635,911
	11,065	15,143	41,647	7,144	59,285	1,635,944
	11,065	16,486	40,304	7,144	59,285	1,636,759
	11,065	16,486	41,647	5,801	59,285	1,634,840
$D_{0,i}$	11,883	17,825	35,650	9,507	59,417	1,628,701
$D_{0,i}^{e^{\mu_i T}}$	11,000	16,918	39,312	7,766	59,286	1,636,089

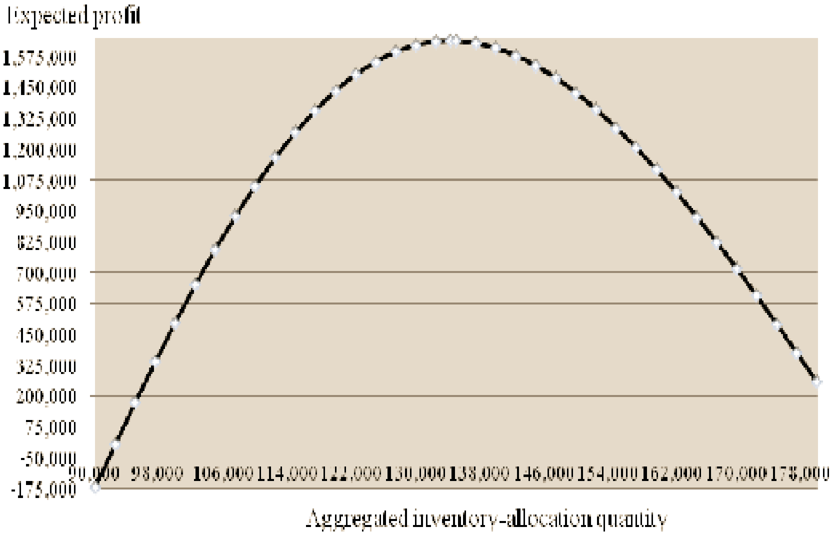

Figure 1: Variation in expected profits over a given range of inventory quantities.

Additionally, this study performs sensitivity analysis for key parameters. Among the model parameters, demand volatility, which is commonly measured in terms of the standard deviation of demand growth rate, indicates uncertain future market demand and is crucial in the inventory-allocation decision. As generally realized, since demand volatility often negatively impacts expected profits, in practice vendors wish to reduce demand volatility in order for improving their profitability. On the other hand, volatility of aggregate demand potentially projects the competitive intensity for a given perishable item, and in accordance with Eq. (2.11) this competitive intensity can be diminished by three approaches, namely increasing the number of retailers, decreasing demand volatility of individual retailer, and reducing demand covariance between retailers. Among others, one practical tactic for reducing aggregate demand volatility is to decrease demand covariance between retailers by encouraging retail competition and had better attain a negative demand correlation.

More specifically, aggregate demand volatility is not simply a weighted average of individual demand volatility except that overall pairs of retailer exhibit perfect positive correlation between demands each other. Aggregate demand volatility also considers how the demands of multiple retailers are subject to co-variance such that aggregate demand volatility for a group of retailers is generally smaller than the weighted average of individual demand volatility. This is the advantage of diversification. In this numerical instance, for example, the volatility of aggregate demand is 0.2875, while the simple weighted average of individual demand volatility is considerably higher at 0.3891, suggesting that expected profit only reaches \$1,145,356 and lowers 30% under this volatility level. To conclude, increases in number of retailers and decreases in demand correlation between retailers are two practicable and effective strategies that can ease aggregate demand volatility for vendors.

To better understand the volatility of aggregate demand, this study performs detailed sensitivity analysis against demand volatility and sets the value of the standard deviation (volatility) of aggregate demand ranging from 5% to 95%, within which range it changes in 5% increments, to determine the corresponding optimal inventory quantity, inventory-allocation decision, and maximal expected profit for vendor. Table 3 lists the numerical results for the specified range of volatilities of aggregate demand. The Table obviously shows that the maximal expected profit gradually declines with increasing volatility, and even becomes negative when volatility approaches 65%. Definitely, there is no sense doing business under such circumstances, but it is unusual for demand volatility to be so high in practice. On average, a 5% increment in volatility incurs a loss of 243,647 (or say 24.9311%) in maximal expected profit. Unsurprisingly, the volatility negatively impacts vendor profit.

Additionally, for reasons of perception, Figure 2 also shows the results of sensitivity analysis originated from volatile and uncertain aggregate demand. The profile also reveals a clear divergence between optimal inventory quantity and expected profit in response to changes in demand volatility. The optimal inventory quantity initially increases with volatility, meaning vendors should extend production/order to prevent expensive losses associated with shortage units in circumstances of increased aggregate demand volatility. Remarkably, a transition point appears in the region of 30% of volatility, and the optimal inventory quantity subsequently decreases with volatility, suggesting that in the present circumstances, the potential loss incurred from unsold units exceeds the shortage cost. Additionally, Figure 2 also reveals that the increase in volatility of aggregate demand continuously and negatively influences expected profit. Being consistent with above observations, high demand volatility generally negatively impacts vendor profitability, and in this condition expanding numbers of new retailers and intensifying competition among retailers to reduce the covariance between demands can help decrease aggregate demand volatility and thus increase expected profit.

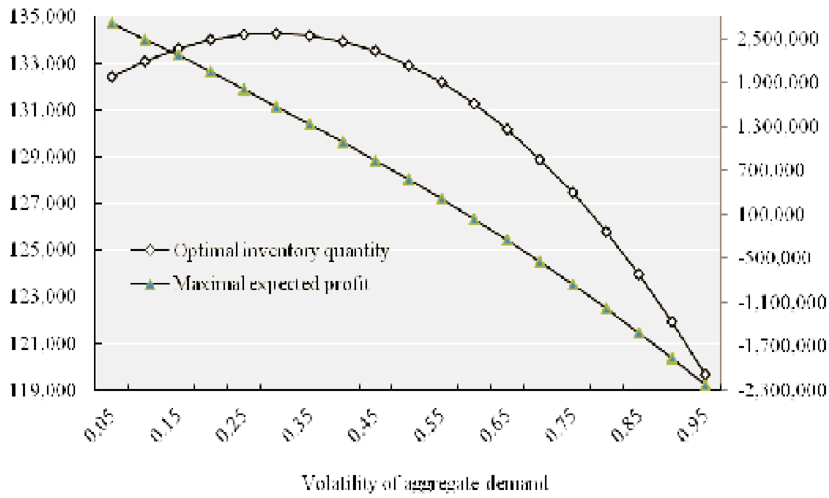

Figure 2: Effect of changes in volatility of aggregate demand on optimal inventory quantity and maximal expected profit.

Table 3: Resulting optimal inventory quantities and maximal expected profits for a given range of volatilities of aggregate demand.

σ_X	Q_S^*	Q_1^*	Q_2^*	Q_3^*	Q_4^*	Q_5^*	$E[R^*]$
0.05	132,420	10,977	16,396	40,755	7,103	57,189	2,714,719
0.10	133,105	11,009	16,430	41,079	7,118	57,468	2,493,902
0.15	133,633	11,034	16,455	41,332	7,130	57,682	2,270,359
0.20	134,006	11,052	16,473	41,512	7,138	57,831	2,043,523
0.25	134,223	11,062	16,483	41,617	7,143	57,918	1,812,807
0.30	134,284	11,065	16,486	41,647	7,144	57,942	1,577,603
0.35	134,187	11,060	16,481	41,600	7,142	57,903	1,337,280
0.40	133,931	11,048	16,469	41,476	7,136	57,801	1,091,179
0.45	133,514	11,029	16,449	41,275	7,127	57,634	838,608
0.50	132,933	11,001	16,421	40,997	7,114	57,398	578,840
0.55	132,185	10,966	16,385	40,644	7,097	57,093	311,110
0.60	131,267	10,922	16,339	40,216	7,076	56,714	34,606
0.65	130,175	10,869	16,284	39,713	7,051	56,258	-251,535
0.70	128,905	10,806	16,219	39,136	7,020	55,723	-548,230
0.75	127,451	10,734	16,143	38,482	6,985	55,106	-856,461
0.80	125,808	10,652	16,057	37,749	6,945	54,405	-1,177,279
0.85	123,970	10,559	15,959	36,932	6,900	53,620	-1,511,811
0.90	121,929	10,456	15,850	36,020	6,850	52,753	-1,861,268
0.95	119,674	10,343	15,731	34,996	6,795	51,809	-2,226,958
Average	130,400	10,876(0.0834)	16,290(0.1250)	39,852(0.3054)	7,053(0.0541)	56,329(0.4320)	

Another manageable crucial parameter is adjustment cost and thus this study also observes and analyzes its sensitivity. To this end, assuming the remaining parameters are constant, adjustment cost of each retailer varies in sequence with a percentage ranging from -50% to 50%, and changing in increments of 20%. Table 4 lists the variant effects of adjustment cost changes for every retailer on the optimal inventory quantity, inventory-allocation decision, and expected profit. Also, the bold numbers in Table 4 indicate the consequences when adjustment costs for all five retailers simultaneously change at the same rate. To compare the differences in effects among retailers, Figure 3, which is drawn from Table 4, further illustrates the separate results for variability of expected profit of each retailer.

Figure 3 clearly shows that retailers 5 and 4 have respectively the most significant and second most significant effects of adjustment cost on vendor expected profit. Consequently, the profitability could be substantially improved if adjustment costs for retailers 5 and/or 4 are reduced. Additionally, Table 4 also reveals that, as generally expected, both optimal inventory quantity and maximal expected profit constantly and negatively vary with overall adjustment cost, and on average optimal inventory quantity only slightly decreases, by about 0.0318%, while maximal expected profit comparably declines, by about 1.1001% (or say $18,122), given an increase of 20% (or say $1.1) in overall adjustment cost. It can thus be seen that a reduction in adjusting frequency and adjustment cost, especially for more sensitive retailers, also is quite a relevant element to increase profitability.

Table 4: Resulting optimal inventory quantities and maximal expected profits for given variation rates of adjustment cost. Variationrate.

Variation rate	Q_S^*	Q_1^*	Q_2^*	Q_3^*	Q_4^*	Q_5^*	$E[R^*]$
	134,293	11,459	16,467	41,451	7,135	57,781	1,638,242
	134,293	11,046	16,871	41,456	7,136	57,785	1,645,131
-50%	134,357	11,034	16,455	41,332	7,081	56,793	1,640,199
	134,288	11,056	16,477	41,556	7,331	57,868	1,646,991
	134,347	10,949	16,367	40,478	7,143	57,918	1,661,527
	134,392	**11,070**	**16,491**	**41,700**	**7,147**	**57,985**	**1,683,894**
	134,287	11,232	16,478	41,563	7,140	57,873	1,637,712
	134,287	11,057	16,654	41,563	7,140	57,873	1,641,846
-30%	134,321	10,995	16,415	42,459	7,111	57,342	1,638,855
	134,285	11,061	16,482	41,608	7,224	57,910	1,642,969
	134,315	11,006	16,426	41,042	7,116	58,725	1,651,651
	134,348	**11,068**	**16,489**	**41,678**	**7,146**	**57,968**	**1,665,114**
	134,284	11,108	16,484	41,625	7,143	57,924	1,637,202
	134,284	11,063	16,530	41,625	7,143	57,924	1,638,580
-10%	134,294	11,044	16,465	41,885	7,134	57,765	1,637,574
	134,284	11,064	16,485	41,636	7,165	57,933	1,638,955
	134,292	11,048	16,469	41,470	7,136	58,170	1,641,839
	134,305	**11,066**	**16,487**	**41,657**	**7,145**	**57,950**	**1,646,337**
	134,282	11,029	16,488	41,664	7,145	57,956	1,636,701
	134,282	11,066	16,450	41,665	7,145	57,956	1,635,323
10%	134,273	11,083	16,505	41,434	7,153	58,099	1,636,330
	134,283	11,065	16,487	41,655	7,127	57,949	1,634,946
	134,275	11,080	16,501	41,804	7,151	57,740	1,632,070
	134,262	**11,064**	**16,485**	**41,636**	**7,144**	**57,933**	**1,627,564**
	134,281	10,975	16,490	41,691	7,146	57,978	1,636,204
	134,281	11,069	16,394	41,692	7,146	57,979	1,632,070
30%	134,256	11,114	16,536	41,071	7,167	58,367	1,635,130
	134,282	11,067	16,488	41,668	7,101	57,959	1,630,940
	134,262	11,104	16,526	42,070	7,163	57,398	1,622,330
	134,219	**11,062**	**16,483**	**41,615**	**7,143**	**57,916**	**1,608,795**
	134,280	10,934	16,492	41,711	7,147	57,995	1,635,711
	134,280	11,071	16,353	41,713	7,147	57,996	1,628,821
50%	134,243	11,140	16,562	40,775	7,179	58,587	1,633,946
	134,282	11,068	16,489	41,677	7,081	57,967	1,626,934
	134,251	11,124	16,546	42,288	7,172	57,121	1,612,610
	134,179	**11,057**	**16,541**	**41,565**	**7,140**	**57,875**	**1,593,287**

4. Concluding Remarks

This study extends a typical newsvendor model to incorporate vendor inventory-allocation integrated decision in a single-vendor multi-retailer supply chain system, which has been becoming a prevalent channel arrangement in practice. This study endeavors to develop the inventory-allocation integrated decision model from a vendor viewpoint.

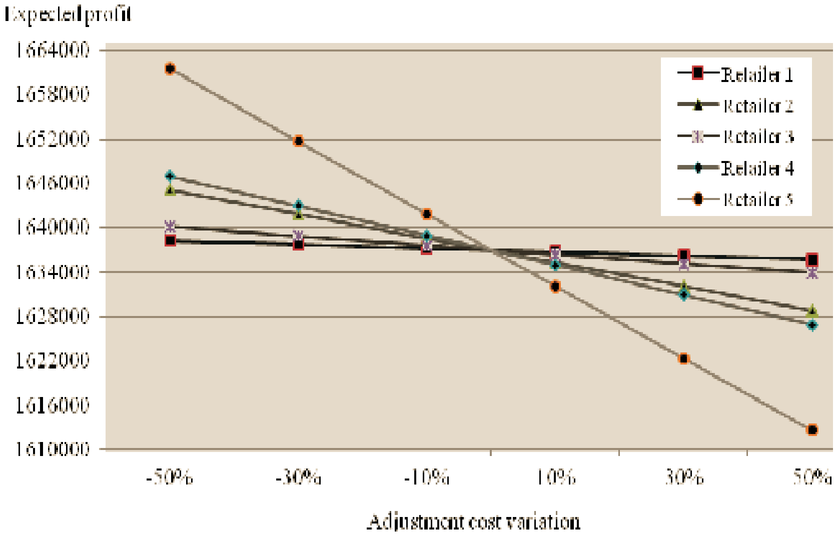

Figure 3: Effect of changes in individual retailer adjustment cost on maximal expected profit.

Therefore, the decision model developed here is applicable to widespread vendor-managed inventory supply chain systems. Moreover, this decision model should be particularly valuable and necessary for perishable items because negligible salvage value often remains on unsold units, excessive shortage costs for stock-out units are punished, and a substantial adjustment cost is frequently incurred for frequent adjustments between retailers due to imprecise allocation. By contract, this study assumes that individual retailer demand for a given perishable item during a selling period exhibits a lognormal distribution, which is considered more reasonable than the familiar normal distribution. Additionally, it is special and notable in this study that Ito process is applied to imitate and model the stochastic shift behavior of market demand and a novel and comprehensive geometric average transformation device is employed to address the problem of non-lognormal aggregate demand.

After some efforts, this study finally develops an effectual and practicable analytical model for optimizing inventory quantity and inventory-allocation decision so as to maximize vendor expected profit during the upcoming selling period. This study takes a plausibly supposed numerical instance to demonstrate that the proposed analytical model can, as expected, solve the optimal inventory quantity and inventory-allocation decision for vendor to earn the maximal expected profit. Additionally, sensitivity analysis is performed for the crucial parameter volatility of aggregate demand and adjustment cost, and reveals some interesting and noticeable managerial insights. In summary, the analytical model presented herein and the experimental findings can help vendors, who trade in perishable items in the case of single-vendor multi-retailer supply chain systems, improve their profitability. The applicable future researches based on the works in this study include, for example, incorporating wholesale and/or retail pricing policies, dual channels composed of direct channel and retailer channel, synchronizing joint replenishment and delivery cycles, allowing for returns policies, involving multiple perishable items, and extending into a multi-vendor multi-retailer supply chain with vendors competition.

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Appendix. Proof of Expected Vendor Profit

Eq. (2.16) can be solved by first solving the four underlying components embraced in the equation, as detailed below, then linking these four components to return to the original expression.

(1). $\int_{Q_S/B+A-1}^{\infty} f(X)dX$ and $\int_{Q_i}^{\infty} f(D_i)dD_i$.

This component can be algebraically derived as follows:

$$\int_{Q_S/B+A-1}^{\infty} f(X)dX = \int_{Q_S/B+A-1}^{\infty} \frac{1}{X}\frac{1}{\sigma_X\sqrt{T}\sqrt{2\pi}} e^{\left[\frac{-(\ln X - E[\ln X])^2}{2\sigma_X^2 T}\right]} dX. \tag{A.1}$$

Let $\ln X = s$, $E(\ln X) = \bar{s}$ and $\sigma_X\sqrt{T} = u$; Eq. (A.1) can then be transformed into the following expression.

$$\int_{Q_S/B+A-1}^{\infty} \frac{1}{X}\frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dX. \tag{A.2}$$

Furthermore, if $w = \frac{(s-\bar{s})}{u}$, then $dw = \frac{1}{uX}dX$ through differentiation. The lower bound of the integral for w is accordingly transformed as follows:

$$= \frac{\ln[(Q_S + AB - B)/B] - E[\ln X]}{u} = \frac{\ln(Q_S + AB - B) - \ln B - \mu_X T}{\sigma_X\sqrt{T}}$$
$$= \frac{\ln[B/(Q_S + AB - B)] + \mu_X T}{\sigma_X\sqrt{T}} = -d_{01}.$$

Again, after applying dw and the lower bound of the integral for w in Eq. (A.2), the equation can be reformulated as

$$\int_{Q_S/B+A-1}^{\infty} \frac{1}{X}\frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dX = \int_{-d_{01}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(w^2/2)} dw = N(d_{01}). \tag{A.3}$$

Similarly, the following expression can also be obtained through the above procedure.

$$\int_{Q_i}^{\infty} f(D_i)dD_i = N(d_{i1}), \tag{A.4}$$

$$d_{i1} = \frac{\ln[D_{0,i}/Q_i] + (\mu_{D_i} - \sigma_{D_i}^2/2)T}{\sigma_{D_i}\sqrt{T}}.$$

(2). $\int_{Q_S/B+A-1}^{\infty} Xf(X)dX$ and $\int_{Q_i}^{\infty} D_i f(D_i)dD_i$.

The component can be deduced in a similar way. First, the component can be expanded as:

$$\int_{Q_S/B+A-1}^{\infty} Xf(X)dX = \int_{Q_S/B+A-1}^{\infty} \frac{1}{\sigma_X\sqrt{T}\sqrt{2\pi}} e^{\frac{-(\ln X - E[\ln X])^2}{2\sigma_X^2 T}} dX. \tag{A.5}$$

The integral can be obtained via the same procedure used for component (1):

$$\begin{aligned}\int_{Q_S/B+A-1}^{\infty} \frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dX &= Ae^{-\ln A} \int_{Q_S/B+A-1}^{\infty} e^{\ln X} \frac{1}{X} \frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dX \\ &= A \int_{Q_S/B+A-1}^{\infty} \frac{1}{X} \frac{1}{u\sqrt{2\pi}} e^{\frac{-[s-(\bar{s}+u^2)]^2}{2u^2}} dX. \end{aligned} \tag{A.6}$$

Let $y = \frac{s-(\bar{s}+u^2)}{u}$, then $dy = \frac{1}{u}ds = \frac{1}{u}d(\ln X) = \frac{1}{uX}dX$ can be obtained. Likewise, the lower bound of the integral for y is transformed as follows:

$$\begin{aligned} &= \frac{s-(\bar{s}+u^2)}{u} = \frac{\ln[(Q_S + AB - B)/B] - (E[\ln X] + \sigma_X^2 T)}{\sigma_X\sqrt{T}} \\ &= \frac{\ln[B/(Q_S + AB - B)/B] - (\mu_X T + \sigma_X^2 T)}{\sigma_X\sqrt{T}} = \frac{\ln[B/(Q_S + AB - B)] + (\mu_X T + \sigma_X^2)T}{\sigma_X\sqrt{T}} \\ &= -d_{02}. \end{aligned}$$

Including dy and the lower bound of the integral for y in Eq. (A.6) yields the following equation:

$$\int_{Q_S/B+A-1}^{\infty} Xf(X)dX = A\int_{-d_{02}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-(y^2/2)} dy = A[1 - N(-d_{02})] = AN(d_{02}). \tag{A.7}$$

Similarly, the deduction of $\int_{Q_i}^{\infty} D_i f(D_i)dD_i$ results in the following.

$$\int_{Q_i}^{\infty} D_i f(D_i)dD_i = D_{0,i}e^{\mu_{D_i}T} N(d_{i2}), \tag{A.8}$$

$$d_{i2} = \frac{\ln[D_{0,i}/Q_i] + (\mu_{D_i} + \sigma_{D_i}^2/2)T}{\sigma_{D_i}\sqrt{T}}.$$

(3). $\int_{0}^{Q_S/B+A-1} f(X)dX$ and $\int_{0}^{Q_i} f(D_i)dD_i$

Because the deduction of this component closely resembles that for component (1), the procedure is not detailed. The following two close-form formulas are also identified:

$$\int_0^{Q_S/B+A-1} f(X)dX = N(-d_{01}) = 1 - N(d_{01}), \tag{A.9}$$

$$\int_0^{Q_i} f(D_i)dD_i = 1 - N(d_{i1}). \tag{A.10}$$

(4). $\int_0^{Q_S/B+A-1} Xf(X)dX$ and $\int_0^{Q_i} D_i f(D_i)dD_i$

Likewise, this component is deduced in much the same manner as component (2). Thus, only the final outcomes are presented, as follows:

$$\int_0^{Q_S/B+A-1} Xf(X)dX = AN(-d_{02}) = A[1 - N(d_{02})], \tag{A.11}$$

$$\int_0^{Q_i} D_i f(D_i)dD_i = D_{0,i}e^{\mu_{D_i}T}[1 - N(d_{i2})]. \tag{A.12}$$

By applying the results of Eqs. (A.3), (A.4), (A.7), (A.8), (A.9), (A.10), (A.11) and (A.12) to Eq. (2.16), the expected vendor profit can thus be solved.

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