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System Reliability Evaluation for an Emergency Department Service System

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Abstract

An emergency department service system provides a medical service to patients who are prioritized for medical care, starting with triage categories and ending with the discharge formalities. In this paper, the emergency department service system is modeled as a fuzzy multi-state network called emergency department service network (EDSN). Each arc is a workstation with a fuzzy multi-state situation because of the vagueness in measuring human capability, and each node is represented as a waiting area. The main aim of this paper is to evaluate the system reliability, that is, the possibility that an emergency department service system can provide necessary treatments to the patients. Since each workstation is fuzzy multi-state, we proposed a method to generate the membership function by utilizing by statistical parameters, such as average and standard deviation. A practical example with statistical data taken from a Taiwan hospital to evaluate the system reliability is demonstrated.

Keywords: Emergency department service network (EDSN), fuzzy multi-state network, system reliability, fuzzy arithmetic.

1. Introduction

The emergency department service system offers comprehensive emergency care and responds to patients within 24 hours. With economic growth and an improvement in living standards, people are increasingly paying attention to health issues. In addition, the shifting age structure has contributed to an increase in patient demand for emergency department service (see Lin and Huang [18]). That is, the visits by elderly patients to the emergency department have grown, as they pursue the desired medical service (see Chen et al. [4]). This has led to crowding in the emergency department, with the number of patients exceeding the capacity of the emergency department to provide medical services. A crowded emergency department not only leads to staff shortage but also a dip in the quality of medical quality provided by the emergency department service system (see Sun et al. [27]). Rossetti et al. [26] established a method to measure the emergency department staff workload and improve their efficiency. At the triage level, there is no appropriate performance index to assess the performance of the emergency department with the uncertain staff workload. For evaluating such the system, the system reliability is defined as the possibility that the available workstations can provide necessary treatments to the patients, and each workstation represents a couple of staff members. Therefore, in this study, we utilize a fuzzy arithmetic approach to evaluate the system reliability to understand the performance of the emergency department.

System reliability can be evaluated in terms of minimal paths (MPs) or minimal cuts (MCs). Many studies have applied the concept of system reliability to many real-life systems, such as node failure (see Lin [10], Lin and Huang [15]). multi-commodity (see Lin [11], Lin [12]), time and budget constraints (see Lin and Chang [13], Lin et al. [17], Lin and Yeh [19], Lin and Yeh [20]), and multiple sinks (see Lin et al. [14], Lin and Huang [16], Lin et al. [22]). Lin and Chang [13] considered reworking action and failure rates of machines in a manufacturing system. Lin and Huang [16] studied a computer system when considering the accuracy rate to evaluate the system reliability. Lin [12] applied the MCs concept to evaluate the system reliability of the transportation system with multi-commodity and cost constraints as features. These studies assume that the probability distribution of each component is given. To further calculate the system reliability, the inclusion-exclusion principle (see Lin [10], Xue [29], Yeh [30], Yeh [31]), the state-space decomposition (see Aven [1], Lin et al. [9], Lin and Yeh [19]), or the Recursive Sum of Disjoint Products (RSDP) (see Lin and Yeh [20], Lin and Yeh [21], Zuo et al. [35]) are adopted. However, the concepts of MPs and MCs are not suitable for the emergency department service system; this is because the emergency department system involves a number of employees, and does not have the exact probability distribution for the capacity of each employee. Hence, the system reliability is not easy to assess when human capacity is considered. In order to overcome this challenge, Chang and Lin [3] utilized fuzzy arithmetic to model the labor-intensive manufacturing network as a fuzzy multi-state network.

Fuzzy set theory emerged in the 1960s, and proposed several concepts, such as the membership function, the intersection of operation, the union of operation, and linguistic variables (see Zadeh [32]). Several studies have proposed membership functions, such as the triangular, trapezoidal, L, and Gaussian functions (see Klir and Yuan [6], Kreinovich et al. [7], Ross [25], zadeh [32]). In particular, the triangular membership function is commonly applied to adaptive-network-based fuzzy inference system and fuzzy analytic hierarchy process (see Jang [5], Mikhailov and Tsvetinov [23]). However, there are some studies in which the triangular membership function curve is generated through statistical analysis. Zhang and Deng [33] proposed a method to construct the triangular membership function with statistical parameters. Similarly, Zhang et al. [34] proposed a method to construct the triangular and Gaussian membership function to evaluate the slope of construction.

Due to the uncertainty in measuring human capability, the capacity of staff, unlike that of a machine, is imprecise. In other words, the human capability may involve judgments based on imprecise information. Hence, the proposed algorithm utilizes the fuzzy arithmetic to calculate the vague value at a workstation, which is managed by a couple of staff members. In fuzzy arithmetic, we generate membership functions by means of the statistics parameters including average and standard deviation, obtained from the statistical data. For emergency department service system, the fuzzy multistate component can be measured by the fuzzy membership function to characterize the lower, normal, and high levels of treatment. Then, the system reliability is then evaluated to learn the capability of the emergency systems.

The main difference between this paper and the previous study (see Chang and Lin [3]) is that the membership functions in this paper are determined according to the lower, average, and upper limits that are related to average and standard deviation to replace the first, second, and third quartiles in Ref (see Chang and Lin [3]). Note that the previous study (see Chang and Lin [3]) determined membership function according to the quartiles determined by historical data. This paper adopts network analysis and fuzzy arithmetic to construct emergency department service system as an emergency department service network (EDSN). In the EDSN, each arc is a workstation and each node is denoted as a waiting area following the workstation. The workstation is characterized by multi-states, such as lower, normal, and high level of treatment. Hence, the EDSN can be regarded as a fuzzy multi-state network. The system reliability is defined as the possibility that all the workstations available can provide necessary treatments to the patients. In addition, a case of emergency department system is adopted to demonstrate the proposed algorithm and present the experiment result. Through the proposed algorithm, the emergency department manager can find out whether or the demand can be met.

2. Problem Modeling

This section applies fuzzy arithmetic to determine the loading state of each workstation in the EDSN. The fuzzy sets and membership are utilized to first assess the workstation reliability. Subsequently, we evaluate the system reliability of the EDSN in term of workstation reliabilities.

Notations

d demand: the number of patients who need treatments in emergency department.

N set of nodes (waiting area)

a_i ith arc (workstation), $i = 1, 2, \dots, 6$.

A $\{a_i \mid i = 1, 2, \dots, 6\}$: set of arcs.

Y $(y_1, y_2, \dots, y_6)$: workstation vector.

- m level of patients by emergency triage.
- L_m average number of patients at triage level m, $m = 1, 2, \ldots, 4$.
- V_m percentage of f patients at triage level m, $m = 1, 2, \ldots, 4$.
- $\bar{x}_i$ average number of patients who can be treated by medical staff at a_i.
- s_i standard deviation of the number of patients who can be treated by medical staff at a_i.
- q_1^i lower limit at a_i.
- q_2^i average value at a_i.
- q_3^i upper limit at a_i..
- G (N, A, Y): a multistate network for an emergency department..
- y_i current number of patients at a_i, $i = 1, 2, \ldots, 6$..
- $\mu_L^i(y_i)$ the membership function of low level treatment under y_i at a_i.
- $\mu_N^i(y_i)$ the membership function of normal level treatment under y_i at a_i.
- $\mu_H^i(y_i)$ the membership function of high level treatment under y_i at a_i.
- $\mu_R^i(y_i)$ the membership function for workstation reliability at a_i.
- R_d system reliability: the probability that the EDSN can satisfy demand d.

The emergency department service system provides urgent care round the clock to patients through many medical resources, such that professional medical staff, comprehensive medical processes, advanced medical equipment, and special administrative system. After being brought into the emergency ward, the patient undergoes several procedures, such as triage category, registration, medical treatment, checking and inspection, and discharge formalities; the procedures are shown in Figure 1. A patient enters the emergency department through one of two modes: walk-in or rescue vehicle. If the patients' condition is critical (level-1), the triage nurse skips the registration step and immediately takes the patient to a treatment room. The other levels of patients follow the emergency department treatment process.

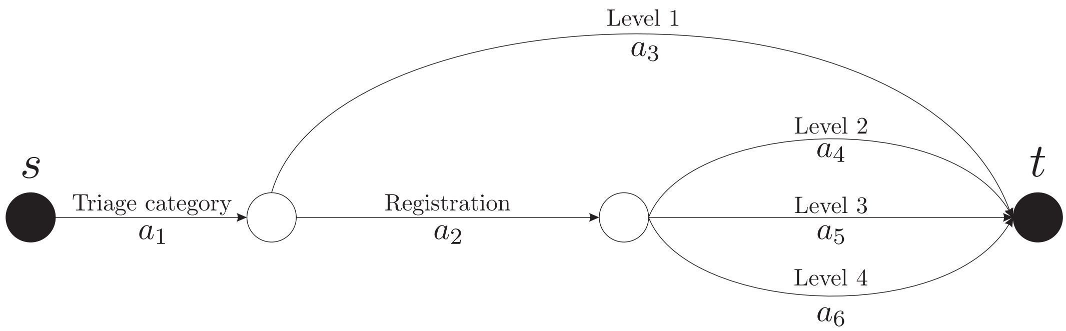

Figure 1: The emergency department treatment process.

Let $G \equiv (N, A, Y)$ denote an EDSN with n arcs (workstations), where N represents the set of nodes (waiting area), $A = \{a_i \mid i = 1, 2, \dots, 6\}$ is the set of arcs (workstations); and $Y = (y_1, y_2, \dots, y_6)$ represents the loading vector which satisfies demand d. If $Y = (y_1, y_2, \dots, y_6)$ is a feasible loading vector, then it should satisfy equations (2.1), (2.2), and (2.3), as follows

$$Y_i = \lceil d \times V_m \rceil, \tag{2.1}$$

$$y_2 = \sum_{i=4}^{6} y_i, \tag{2.2}$$

$$y_1 = y_2 + y_3. \tag{2.3}$$

In this model, considering the uncertainty in measuring human capability, the fuzzy logic technique is used to determine the capacity of the staff. The emergency department service system is service-oriented to ensure that the staff provides the proper care to their patients. The assumptions made in this paper are as follows.

I. The loading state of each arc (workstation) takes linguistic values given by {low level treatment (L), normal treatment (N), high treatment (H)}. The loading state is governed by membership function mapping from the fuzzy set.

II. State possibility is represented by fuzzy values.

III. Each node (waiting area) is perfectly reliable and with infinite capacity.

2.1. Linguistic variable

The fuzzy linguistic variable is identified as low level treatment (L), normal level treatment (N), and high level treatment (H). The input value is mapped into the membership function graph to obtain the performance in the EDSN. The real value that is supplied into the system is converted to linguistic variables. To determine the loading state at a_i by using the fuzzy approach, the three fuzzy variables describing the loading state for y_i are as follow:

$L \equiv$ 'y_i is low level treatment for a_i',

$N \equiv$ 'y_i is normal level treatment for a_i',

$H \equiv$ 'y_i is high level treatment for a_i',

We formulate the membership function for the three fuzzy variables that the membership functions of low, normal, and high level treatment under y_i at the ith workstation are denoted by $\mu_L^i(y_i)$, $\mu_N^i(y_i)$, and $\mu_H^i(y_i)$, respectively. For this emergency department service system, the three membership functions, $\mu_L^i(y_i)$, $\mu_N^i(y_i)$, and $\mu_H^i(y_i)$, are drawn for the corresponding variables. The x-axis represents the parameters of the triangular members while the y-axis represents the possibility range from 0 to 1.0. The membership function is used to represent the linguistic variables that appear in Figure 2.

Figure 2 shows that different types of membership function are employed at each workstation. The membership function of low level treatment ($\mu_L^i(y_i)$) is a L-function;

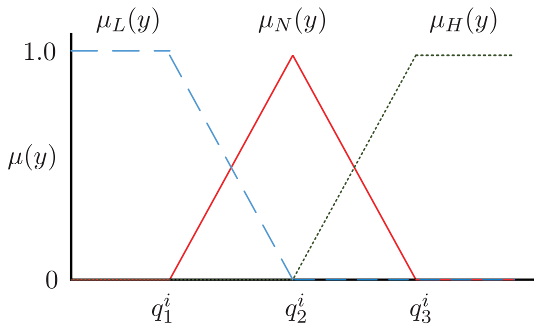

Figure 2: The membership function.

the membership function of normal treatment ($\mu_N^i(y_i)$) is a Triangular function; and the membership of high treatment ($\mu_H^i(y_i)$) is Gamma membership function in a linear way (see Chaira and Ray [2], Pedrycz [24], Tarng et al. [28]). All the membership functions are limited by q_1^i, q_2^i and q_3^i, where the fuzzy set denoted by q_1^i represents the lower limit at a_i; q_2^i represents the average value at a_i; and q_3^i represents the upper limit at a_i.

2.2. Fuzzy membership curve generating

We have focused on the proposed membership function with three parameters — q_1^i, q_2^i and q_3^i. The three parameters are constructed from average $\bar{x}_i$ and standard deviation s_i, which represent the average and the standard deviation, respectively, of the number of patients that can be treated by medical staff at the ith workstation (a_i). Both $\bar{x}_i$ and s_i can easily be obtained from the statistical data. Zhang et al. [34] took a value of $2.5s_i$ to generate the triangular membership function with lower and upper limit. Further, Zhang and Deng [33] utilized a value of $0.4s_i$ to generate the triangular membership function with lower and upper limit. In this paper, we utilize a value of $2.5s_i$ to generate the membership function with lower and upper limit. The three parameters are discussion as follow.

The membership function of lower limit q_1^i is generated by equation (2.4)

$$q_1^i = \max\{\lfloor \bar{x}_i - 2.5s_i \rfloor, 0\}. \tag{2.4}$$

where $\lfloor \bar{x}_i - 2.5s_i \rfloor$ is the biggest integer value that is smaller than or equal to $\bar{x}_i - 2.5s_i$. In general, the value of q_1^i must be larger than 0.

The membership function of the average value of q_2^i is generated by equation (2.5)

$$q_2^i = \lceil \bar{x}_i \rceil. \tag{2.5}$$

The membership function of the upper limit q_3^i is generated by equation (2.6)

$$q_3^i = \lceil \bar{x}_i + 2.5s_i \rceil. \tag{2.6}$$

In this paper, we adopt 2.5 times standard deviation to generate membership function of the lower limit and upper limit. Now that the fuzzy membership curve is a function of a loading state vector Y and depends on three scalar parameters — q_1^i, q_2^i and q_3^i — given by

$$\mu_L^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i > q_2^i \\ (q_2^i - y_i)/q_2^i - q_1^i & \text{if} \quad q_1^i \le y_i \le q_2^i \\ 1 & \text{if} \quad y_i < q_1^i \end{cases}, \tag{2.7}$$

$$\mu_N^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i < q_1^i \text{ or } y_i > q_3^i \\ (q_3^i - y_i)/(q_3^i - q_2^i) & \text{if} \quad q_2^i \le y_i \le q_3^i \\ (y_i - q_1^i)/(q_2^i - q_1^i) & \text{if} \quad q_1^i \le y_i < q_2^i \end{cases}, \tag{2.8}$$

$$\mu_H^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i < q_2^i \\ (y_i - q_2^i)/q_3^i - q_2^i & \text{if} \quad q_2^i \le y_i \le q_3^i \\ 1 & \text{if} \quad y_i > q_3^i \end{cases}. \tag{2.9}$$

The membership function transforms the input values to a $(0, 1)$ scale, and the distribution of the value of membership between the two limits is linear (see Pedrycz [24]). The loading state of a workstation among $\{L, N, H\}$ is assigned on the basis of the maximum value from $\{\mu_L^i(y_i), \mu_N^i(y_i), \mu_H^i(y_i)\}$.

2.3. Operation on fuzzy sets

The system reliability is generally defined as $Pr\{X \mid X \ge Y\} = Pr\{X \mid X \ge (y_1, y_2, \ldots, y_n)\} = Pr\{X \mid X \ge (y_1, y_2, \ldots, y_n)\} = Pr\{\{x_1 \ge y_1\} \bigcap \{x_2 \ge y_2\} \bigcap \cdots \bigcap \{x_n \ge y_n\}\}$ in previous studies [10, 11]. In this study, we are concerned with the T-norm functions for computing system reliability, the possibility that the available workstations can satisfy demand in a fuzzy environment (see Chang and Lin [3]). Assume that there are two fuzzy sets, A and B, representing two workstations. A T-norm is a function $T : [0, 1] \times [0, 1] = [0, 1]$ that transforms the membership functions of fuzzy sets A and B into membership function for the intersection of A and B (see Klir and Yuan [6]). There has been study on T-norm, including those that consider Minimum T-norm and Algebraic product T-norm (see Zadeh [32]). Both types of T-norm are defined below.

The Minimum T-norm function is:

$$t(a, b) = \min[a, b]. \tag{2.10}$$

The Algebraic product T-norm function is:

$$t(a, b) = ab. \tag{2.11}$$

2.4. System reliability

The system reliability denoted by R_d is defined as the possibility that all the available workstations can provide necessary treatments to the patients. For evaluating the system reliability, the membership function for the reliability of workstation a_i is denoted by $\mu_R^i(y_i)$ and defined as

$$\mu_R^i(y_i) = \begin{cases} \mu_L^i(y_i) + \mu_N^i(y_i) & \text{if} \quad \max(\mu_L^i(y_i) + \mu_N^i(y_i)) \geq \mu_H^i(y_i) \\ c(\mu_H^i(y_i)) & \text{otherwise} \end{cases}. \tag{2.12}$$

where $c(\mu_H^i(y_i)) = 1 - \mu_H^i(y_i)$ which is a fuzzy complement.

As mentioned in the previous subsections, the concept of T-norm is applied in this membership function for the system reliability as follows:

$$R_d = \mu_R(Y) = t(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^6(y_6)). \tag{2.13}$$

By applying equation (2.10) of Minimum T-norms, the system reliability can obtain as

$$\mu_R(Y) = t(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^6(y_6)) = \min(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^6(y_6)). \tag{2.14}$$

By applying equation (2.11) of Algebraic product T-norms, the system reliability can be obtained as

$$\mu_R(Y) = t(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^7(y_7)) = (\mu_R^1(y_1) \times \mu_R^2(y_2) \times \cdots \times \mu_R^6(y_6)). \tag{2.15}$$

2.5. Algorithm for system reliability

The following are the steps of the algorithm proposed to evaluate system reliability.

Input: $\bar{x}_i$, s_i, $Y = (y_1, y_2, \ldots, y_6)$.

Step 1. Generate the three parameters of the membership function for each workstation

1.1. Find the lower limit of the membership function

$$q_1^i = \max\{\lfloor \bar{x}_i - 2.5s_i \rfloor, 0\}.$$

1.2. Find the average value of the membership function

$$q_2^i = \lceil \bar{x}_i \rceil.$$

1.3. Find the upper limit of the membership function

$$q_3^i = \lceil \bar{x}_i - 2.5s_i \rceil.$$

Step 2. Compare the loading state of each workstation with the value of the membership function

$$\mu_L^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i > q_2^i \\ (q_2^i - y_i)/q_2^i - q_1^i & \text{if} \quad q_1^i \leq y_i \leq q_2^i \\ 1 & \text{if} \quad y_i < q_1^i \end{cases},$$

$$\mu_N^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i < q_1^i \text{ or } y_i > q_3^i \\ (q_3^i - y_i)/(q_3^i - q_2^i) & \text{if} \quad q_2^i \le y_i \le q_3^i \\ (y_i - q_1^i)/(q_2^i - q_1^i) & \text{if} \quad q_1^i \le y_i \le q_2^i \end{cases},$$

$$\mu_H^i(y_i) = \begin{cases} 0 & \text{if} \quad y_i < q_2^i \\ (y_i - q_2^i)/q_3^i - q_2^i & \text{if} \quad q_2^i \le y_i \le q_3^i \\ 1 & \text{if} \quad y_i > q_3^i \end{cases}.$$

The loading state $\{L, N, H\}$ at a_i is determined as $\text{Max}\{\mu_L^i(y_i), \mu_N^i(y_i), \mu_H^i(y_i)\}$, which is the maximum value among $\{\mu_L^i(y_i), \mu_N^i(y_i), \mu_H^i(y_i)\}$.

Step 3. Calculate the workstation reliability for each workstation

$$\mu_R^i(y_i) = \begin{cases} \mu_L^i(y_i) + \mu_N^i(y_i) & \text{if} \quad \max(\mu_L^i(y_i) + \mu_N^i(y_i)) \ge \mu_H^i(y_i) \\ c(\mu_H^i(y_i)) & \text{otherwise} \end{cases}.$$

Step 4. Calculate the system reliability for the EDSN by equations (2.16) and (2.17)

$$\text{Minimum } T\text{-norms } \mu_R(Y) = \min(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^6(y_6)) \tag{2.16}$$

$$\text{Algebraic product } T\text{-norms } \mu_R(Y) = (\mu_R^1(y_1) \times \mu_R^2(y_2) \times \cdots \times \mu_R^6(y_6)). \tag{2.17}$$

Output: System reliability $R_d = \mu_R(Y)$.

The system reliability can be derived through fuzzy operations with Minimum T-norm or Algebraic product T-norm.

3. Case-based Experiments

In order to demonstrate the proposed algorithm, a Taiwan emergency department service system with 4-level triage is presented to evaluate system reliability.

3.1. The emergency department service system with 4-level triage

An emergency department service system in South Taiwan had recorded data for the past 3 years (see Lian [8]). The network with triage 4-level for the system is shown in Figure 1. The important attribute of the percentage of patients at a specific triage level, (V_m), is obtained from the data on the average number of patient listed in Table 1. The average of the patient and standard deviation at each workstation are listed in Table 2. Consider a emergency department service system must satisfy demand $d = 4500$ patients per month, the loading vector $Y = (4501, 4357, 144, 1467, 2866, 24)$ is obtained from equations (2.1), (2.2), and (2.3). After obtained the loading vector, the system reliability can be evaluated as follows.

Table 1: The percentage of average number of patient.

Level l	Average number of patient L_m	The percentage of average number of patient V_m
1	141.46	3%
2	1442.65	32.6%
3	2818.06	63.67%
4	23.63	0.53%

Table 2: The average and standard deviation of the patients (Unit: patients / month).

	a_1	a_2	a_3	a_4	a_5	a_6
$\bar{x}_i$	4425.78	4284.32	141.46	1442.65	2818.05	23.62
s_i	569.51	548.51	44.94	172.83	475.93	26.83

Step 1. Generate three parameters of the membership function for each workstation

1.1. Find the lower limit of the membership function

$$q_1^1 = \max\{\lfloor x_1 - 2.5s_1 \rfloor, 0\} = \max\{\lfloor 4425.78 - 1423.78 \rfloor, 0\} = 3003,$$
$$q_1^2 = \max\{\lfloor x_2 - 2.5s_2 \rfloor, 0\} = \max\{\lfloor 4284.32 - 1371.28 \rfloor, 0\} = 2914,$$
$$q_1^3 = \max\{\lfloor x_3 - 2.5s_3 \rfloor, 0\} = \max\{\lfloor 141.46 - 112.35 \rfloor, 0\} = 29,$$
$$q_1^4 = \max\{\lfloor x_4 - 2.5s_4 \rfloor, 0\} = \max\{\lfloor 1442.65 - 432.075 \rfloor, 0\} = 1010,$$
$$q_1^5 = \max\{\lfloor x_5 - 2.5s_5 \rfloor, 0\} = \max\{\lfloor 2818.05 - 1189.825 \rfloor, 0\} = 1628,$$
$$q_1^6 = \max\{\lfloor x_6 - 2.5s_6 \rfloor, 0\} = \max\{\lfloor 23.62 - 67.08 \rfloor, 0\} = 0.$$

1.2. Find the average value of the membership function

$$q_2^1 = \lceil \bar{x}_1 \rceil = 4426,$$
$$q_2^2 = \lceil \bar{x}_2 \rceil = 4285,$$
$$q_2^3 = \lceil \bar{x}_3 \rceil = 142,$$
$$q_2^4 = \lceil \bar{x}_4 \rceil = 1443,$$
$$q_2^5 = \lceil \bar{x}_5 \rceil = 2819,$$
$$q_2^6 = \lceil \bar{x}_6 \rceil = 24.$$

1.3. Find the upper limit of the membership function

$$q_3^1 = \lceil \bar{x}_1 + 2.5s_1 \rceil = \lceil 4425.78 + 1423.78 \rceil = 5849,$$
$$q_3^2 = \lceil \bar{x}_2 + 2.5s_2 \rceil = \lceil 4284.32 + 1371.28 \rceil = 5655,$$
$$q_3^3 = \lceil \bar{x}_3 + 2.5s_3 \rceil = \lceil 141.46 + 112.35 \rceil = 254,$$
$$q_3^4 = \lceil \bar{x}_4 + 2.5s_4 \rceil = \lceil 1422.65 + 432.075 \rceil = 1855,$$

$$q_3^5 = \lceil \bar{x}_5 + 2.5s_5 \rceil = \lceil 2818.05 + 1189.825 \rceil = 4008,$$
$$q_3^6 = \lceil \bar{x}_6 + 2.5s_6 \rceil = \lceil 23.62 + 67.08 \rceil = 90.$$

Step 2. Check the loading state of each workstation with the value of the membership function

$$\mu_L^1(4501) = 0 \text{ (note: } y_1 = 4501 > q_2^1 = 4426).$$
$$\mu_N^1(4501) = (q_3^1 - y_1)/(q_3^1 - q_2^1) = (5849 - 4501)/(5849 - 4426) = 0.9473,$$
$$\mu_H^1(4501) = (y_1 - q_2^1)/(q_3^1 - q_2^1) = (4501 - 4426)/(5849 - 4426) = 0.0527.$$

Since $\mu_N^1(4501) = 0.9473$ is the maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_1.

$$\mu_L^2(4357) = 0 \text{ (note: } y_2 = 4501 > q_2^2 = 4285).$$
$$\mu_N^2(4357) = (q_3^2 - y_2)/(q_3^2 - q_2^2) = (5655 - 4357)/(5655 - 4285) = 0.9474,$$
$$\mu_H^2(4357) = (y_2 - q_2^1)/(q_3^1 - q_2^1) = (43574285)/(56554285) = 0.0526.$$

Since $\mu_N^2(4357) = 0.9474$ is the maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_2.

$$\mu_L^3(144) = 0 \text{ (note: } y_3 = 144 > q_2^2 = 142).$$
$$\mu_N^3(144) = (q_3^3 - y_3)/(q_3^3 - q_2^3) = (253 - 144)/(253 - 142) = 0.982,$$
$$\mu_H^3(144) = (y_3 - q_2^3)/(q_3^3 - q_2^3) = (144 - 142)/(253 - 142) = 0.018.$$

Since $\mu_N^3(144) = 0.982$ is the maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_3.

$$\mu_L^4(1467) = 0 \text{ (note: } y_4 = 1467 > q_2^4 = 1443).$$
$$\mu_N^4(1467) = (q_3^4 - y_4)/(q_3^4 - q_2^4) = (1874 - 1467)/(1874 - 1443) = 0.9443,$$
$$\mu_H^4(1467) = (y_4 - q_2^4)/(q_3^4 - q_2^4) = (144 - 142)/(1467 - 1443)/(1874 - 1443) = 0.0557.$$

Since $\mu_N^4(1467) = 0.9443$ is maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_4.

$$\mu_L^5(2866) = 0 \text{ (note: } y_5 = 2866 > q_2^5 = 2819).$$
$$\mu_N^5(2866) = (q_3^4 - y_4)/(q_3^4 - q_2^4) = (4007 - 2866)/(4007 - 2819) = 0.9604,$$
$$\mu_H^5(2866) = (y_5 - q_2^4)/(q_3^4 - q_2^4) = (2866 - 2817)/(4007 - 2819) = 0.0396.$$

Since $\mu_N^5(2866) = 0.9604$ is maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_5.

$$\mu_L^6(24) = 0.$$

$$\mu_N v(24) = (q_3^6 - y_6)/(q_3^6 - q_2^6) = (90 - 24)/(90 - 24) = 1,$$
$$\mu_H v(24) = (y_6 - q_2^6)/(q_3^6 - q_2^6) = (24 - 24)/(90 - 24) = 0.$$

Since $\mu_N^6(24) = 1$ is maximum among the value of the three membership functions, the workstation state is normal treatment (N) at a_6.

Step 3. Calculate the workstation reliability for each workstation

$$\mu_R^1(4501) = \mu_L^1(4501) + \mu_N^1(4501) = 0 + 0.9473,$$
$$\mu_R^2(4357) = \mu_L^2(4357) + \mu_N^2(4357) = 0 + 0.9474,$$
$$\mu_R^3(144) = \mu_L^3(144) + \mu_N^3(144) = 0 + 0.982,$$
$$\mu_R^4(1467) = \mu_L^4(1467) + \mu_N^4(1467) = 0 + 0.9443,$$
$$\mu_R^5(2866) = \mu_L^5(2866) + \mu_N^5(2866) = 0 + 0.9604,$$
$$\mu_R^6(24) = \mu_L^6(24) + \mu_N^6(24) = 0 + 1.$$

Step 4. Calculate the system reliability for the EDSN

Minimum T-norm:

$$\begin{aligned}\mu_R(Y) &= \min(\mu_R^1(y_1), \mu_R^2(y_2), \ldots, \mu_R^6(y_6)) \\ &= \min(0.9473, 0.9474, 0.982, 0.9443, 0.9604, 1) \\ &= 0.9443.\end{aligned}$$

Algebraic product T-norm:

$$\begin{aligned}\mu_R(Y) &= \min(\mu_R^1(y_1) \times \mu_R^2(y_2) \times \cdots \times \mu_R^6(y_6)) \\ &= (0.9473 \times 0.9474 \times 0.982 \times 0.9443 \times 0.9604 \times 1) \\ &= 0.7993.\end{aligned}$$

Output: System reliability $R_d = \mu_R(Y) = 0.9443$ with Minimum T-norms and $R_d = \mu_R(Y) = 0.7993$ with Algebraic product T-norms.

The system reliability R_d can bring some managerial implications for decision-making from the managers. For instance, under $d = 4500$ in this case, the managers can discover the overall system capability to decide whether the system has to be improved/maintained or not in the future. Besides, focusing on each workstation reliability $\mu_R^i(y_i)$, the managers can find out the bottleneck in the emergency department and regard it as the first priority for the following improvement or maintenance. In this case, the bottleneck in the emergency department is a_4. Therefore, the increase of the capacity in a_4 will be the most significant improvement for the system reliability among all the workstations.

The results of the above calculation are summarized in Table 3.

Table 3: The workstation state for $Y = (4501, 4357, 144, 1467, 2866, 24)$.

	q_1^i	q_2^i	q_3^i	$\mu_L^i(y_i)$	$\mu_N^i(y_i)$	$\mu_H^i(y_i)$	State	$\mu_R^i(y_i)$
a_1	3003	4426	5849	0	0.9473	0.0527	N	0.9473
a_2	2914	4285	5655	0	0.9474	0.0526	N	0.9474
a_3	30	142	253	0	0.9820	0.0180	N	0.9820
a_4	1011	1443	1874	0	0.9443	0.0557	N	0.9443
a_5	1629	2819	4007	0	0.9604	0.0396	N	0.9604
a_6	0	24	90	0	1.0000	0.0000	N	1.0000

4. Sensitivity Analysis

We discuss the system reliability for a range of demand levels, given by $d = \{3184, 3223, 3249\}$, and assessed by the proposed algorithm. Table 4 provides the loading and reliability for each workstation; the reliability of each workstation decreases slightly because of the greater burden placed on it. Figure 3 shows the system reliability with Minimum and Algebraic product for $d = \{3184, 3223, 3249\}$. We first select $2.5s_i$ to evaluate the system reliability of the EDSN. From the results, we can conclude that the system reliability with Minimum t-norm operation is higher than that with Algebraic product t-norm operation. Then, suppose that the average demand is 3223 (i.e., $d = 3223$). Under $d = 3223$, the system reliability declines low down 50%. From the system quality viewpoint, the managers have to improve the emergency department immediately. Afterward, the improvement directions could be understood by the minimum workstation reliability. Another critical issue is the standard deviation. Thus, we discuss the impacts by varying the difference from 2.0 to 3.0si to observe the system reliability with Algebraic product, the lower limit of the membership function, the average value of membership function, and the upper limit of the membership function. The results are summarized in Table 5 and Figure 4. The system reliability rises significantly as standard deviation increases from 2.0 to 3.0 for $Y = (4501, 4357, 144, 1467, 2866, 24)$. While

Table 4: Loading and state under different levels.

Demand		a_1	a_2	a_3	a_4	a_5	a_6	R_d: Minimum t-norm	R_d: Algebraic product t-norm
3184	y_i	4353	4299	54	1056	3199	44	0.7335	0.5454
	$\mu_R^i(y_i)$	1	0.9915	1	1	0.7335	0.75		
3223	y_i	4406	4351	55	1068	3238	45	0.7062	0.4999
	$\mu_R^i(y_i)$	1	0.9599	1	1	0.7062	0.7375		
3249	y_i	4442	4387	55	1077	3265	45	0.6872	0.4709
	$\mu_R^i(y_i)$	0.9906	0.9380	1	1	0.6872	0.7375		

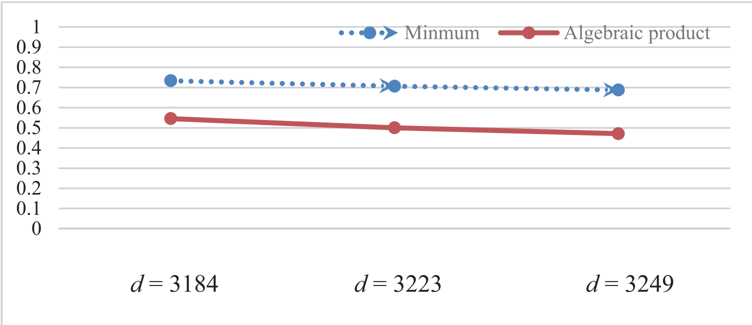

Figure 3: The system reliability with Minimum and Algebraic product for $d = \{3184, 3223, 3249\}$.

the times of standard deviation increases, widths between the lower and upper limits rise, meaning that the measure of human capability become more flexible. Then, the workstation reliability on a specific workstation increases as well. Hence, the sensitivity analysis of standard deviation shown in the experiments is necessary for determining the suitable standard deviation.

Table 5: The numerical result for $Y = (4501, 4357, 144, 1467, 2866, 24)$.

Times	R_d with Algebraic product	The Lower Limit Of Membership Function	The Average Value Of Membership Function	The Upper Limit Of Membership Function
2.0	0.75450085	[3287,3188,52,1097,1867,0]	[4426,4285,142,1443,2819,24]	[5564,5381,231,1788,3769,77]
2.1	0.764914313	[3230,3133,48,1080,1819,0]	[4426,4285,142,1443,2819,24]	[5621,5436,235,1805,3817,79]
2.2	0.77465082	[3173,3078,43,1063,1772,0]	[4426,4285,142,1443,2819,24]	[5678,5491,240,1822,3865,82]
2.3	0.7835218	[3116,3023,39,1046,1724,0]	[4426,4285,142,1443,2819,24]	[5735,5545,244,1840,3912,85]
2.4	0.791812555	[3059,2968,34,1028,1676,0]	[4426,4285,142,1443,2819,24]	[5792,5600,249,1857,3960,88]
2.5	0.799335276	[3003,2914,30,1011,1629,0]	[4426,4285,142,1443,2819,24]	[5849,5655,253,1874,4007,90]
2.6	0.806585436	[2946,2859,25,994,1581,0]	[4426,4285,142,1443,2819,24]	[5906,5710,258,1892,4055,93]
2.7	0.813122979	[2889,2804,21,977,1534,0]	[4426,4285,142,1443,2819,24]	[5963,5765,262,1909,4103,96]
2.8	0.819318997	[2832,2749,16,959,1486,0]	[4426,4285,142,1443,2819,24]	[6020,5820,267,1926,4150,98]
2.9	0.825014803	[2775,2694,12,942,1438,0]	[4426,4285,142,1443,2819,24]	[6077,5874,271,1943,4198,101]
3.0	0.830539151	[2718,2639,7,925,1391,0]	[4426,4285,142,1443,2819,24]	[6134,5929,276,1961,4245,104]

5. Conclusions

To know the capability of an emergency department with uncertain human capability is critical for the managers and even the patients. Hence, the paper mainly contributes to modeling an emergency department, with the fuzzy multi-state representation of the workstations' load to reflect the actual situation of the EDSN. Further, an algorithm based on the fuzzy arithmetic is developed to evaluate the system reliability. In fuzzy arithmetic, the membership function can be generated in terms of the average or standard

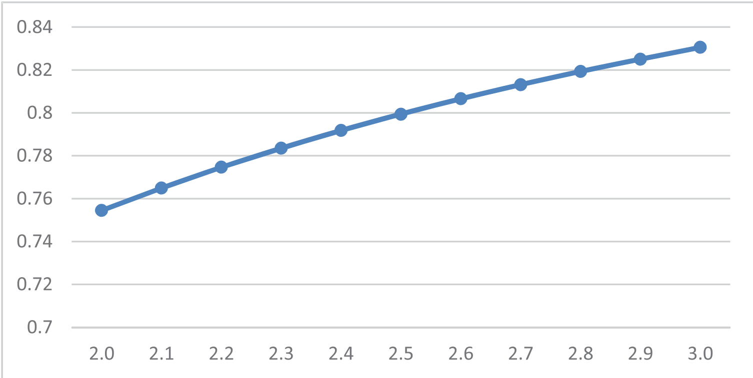

Figure 4: The system reliability with 2.0 to 3.0s_i.

deviation of the data. From the management perspective of the emergency department, system reliability gained from the proposed algorithm is a useful tool that can help the emergency department manager make decisions.

The values of standard deviation and demand both have a significant impact on the system reliability. When the demand increases from 3184 to 3249 units, the system reliability drops from 0.5454 to 0.4709. The system reliability is 0.7545 and 0.8305 when the fuzzy membership curve generated by 2.0 and 3.0s_i, respectively. Thus, Figures 4 and 5 depict the different impact of variation in demand and standard deviation on system reliability. In the paper, the proposed algorithm with fuzzy arithmetic provides sufficient information for the managers of the emergency department to learn the capability of the emergency department. In order to evaluate the system reliability, the first hard mission is to derive the appropriate membership functions and the lower and upper limits with reasonable data. Besides, aside from only affording the system reliability, optimization of the system reliability might be the main priority for the emergency department. Hence, future researchers can focus on the optimization of the system reliability as a topic for deeper insight.

References

[1] Aven, T. (1985). *Reliability evaluation of multistate systems with multistate components*, IEEE Transactions on reliability, Vol.34, 473-479.

[2] Chaira, T. and Ray, A. K. (2003). *Segmentation using fuzzy divergence*, Pattern Recognition Letters, Vol. 24, 1837-1844.

[3] Chang, P. C., and Lin, Y. K. (2015). *Fuzzy-based system reliability of a labour-intensive manufacturing network with repair*, International Journal of Production Research, Vol.53, 1980-1995.

[4] Chen, Y. C., Hwang, S. J., Lai, H. Y., Chen, T. J., Lin, M. H., Chen, L. K. and Lee, C. H. (2009). *Potentially inappropriate medication for emergency department visits by elderly patients in Taiwan*, Pharmacoepidemiology and drug safety, Vol.18, 53-61.

[5] Jang, J. S., (1993). *ANFIS: adaptive-network-based fuzzy inference system*, IEEE Transactions on Systems, man, and Cybernetics, Vol.23, 665-685.

[6] Klir, G. J. and Yuan, B. (1995). Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall, United States.

[7] Kreinovich, V., Quintana, C. and Reznik, L. (1992). *Gaussian membership functions are most adequate in representing uncertainty in measurements*, Proceedings of NAFIPS, Vol.92, 15-17.

[8] Lian, S. Y. (2006). *The Analysis of Emergency Department Utilization under Implementing Different Point-values of Fees*, School of Health Care Administration, National Sun Yat-Sen University, Master thesis. (in Chinese)

[9] Lin, J. S., Jane, C. C. and Yuan, J. (1995). *On reliability evaluation of a capacitated-flow network in terms of minimal pathsets*, Networks, Vol.25, 131-138.

[10] Lin, Y. K. (2007a). *Reliability evaluation for an information network with node failure under cost constraint*, IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, Vol.37, 180-188.

[11] Lin, Y. K. (2007b). *System reliability of a limited-flow network in multicommodity case*, IEEE Transactions on Reliability, Vol.56, 17-25.

[12] Lin, Y. K. (2012). *Performance evaluation for a transportation system in stochastic case*, Computers & Operations Research, Vol.39, 1901-1908.

[13] Lin, Y. K. and Chang, P. C. (2012). *Evaluation of system reliabilities for a maintainable stochastic-flow network*, IEEE Transactions on Reliability, Vol.61, 398-409.

[14] Lin, Y. K., Chang, P. C. and Chen, J. C. (2012). *Reliability evaluation for a waste-reduction parallel-line manufacturing system*, Journal of Cleaner Production, Vol.35, 93-101.

[15] Lin, Y. K. and Huang, C. F. (2013). *Backup reliability of stochastic imperfect-node computer networks subject to packet accuracy rate and time constraints*, International Journal of Computer Mathematics, Vol.90, 457-474.

[16] Lin, Y. K., and Huang, C. F. (2014). *Reliability assessment of a stochastic node-failure network with multiple sinks under tolerable error rate*, International Journal of Computer Mathematics, Vol.91, 819-833.

[17] Lin, Y. K., Huang, C. F., Yeng, L. C. L. and Cho, Y. L. (2017). *Project Reliability Interval for a Stochastic Project Network Subject to Time and Budget Constraints*, IEEE Transactions on Reliability, Vol.66, 689-699.

[18] Lin, Y. Y. and Huang, C. S. (2015). *Aging in Taiwan: Building a society for active aging and aging in place*, The Gerontologist, Vol.56, 176-183.

[19] Lin, Y. K. and Yeh, C. T. (2010). *Evaluation of optimal network reliability under components-assignments subject to a transmission budget*, IEEE Transactions on Reliability, Vol.59, 539-550.

[20] Lin, Y. K. and Yeh, C. T. (2011a). *Computer network reliability optimization under double-resource assignments subject to a transmission budget*, Information Sciences, Vol.181, 582-599.

[21] Lin, Y. K. and Yeh, C. T. (2011b). *Maximal network reliability with optimal transmission line assignment for stochastic electric power network via genetic algorithms*, Applied Soft Computing, Vol.11, 2714-2724.

[22] Lin, Y. K., Yeh, C. T., and Huang, C. F. (2017). *Reliability assessment of a multistate freight network for perishable merchandise with multiple suppliers and buyers*, International Journal of Systems Science, Vol.48, 74-83.

[23] Mikhailov, L. and Tsvetinov, P. (2004). *Evaluation of services using a fuzzy analytic hierarchy process*, Applied Soft Computing, Vol.5, 23-33.

[24] Pedrycz, W. (1994). *Why triangular membership functions?* Fuzzy sets and Systems, Vol.64, 21-30.

[25] Ross, T. J. (2009). Fuzzy Logic with Engineering Applications, John Wiley & Sons, United States: NY.

[26] Rossetti, A. C., Gaidzinski, R. R. and Fugulin, F. M. T. (2013). *Nursing workload in the emergency department: a methodological proposal*, Revista Latino-Americana de Enfermagem, Vol.21, 225-232.

[27] Sun, B. C., Hsia, R. Y., Weiss, R. E., Zingmond, D., Liang, L. J., Han, W. and Asch, S. M. (2013). *Effect of emergency department crowding on outcomes of admitted patients*, Annals of Emergency Medicine, Vol.61, 605-611.

[28] Tarng, Y. S., Tseng, C. M., and Chung, L. K. (1997). *A fuzzy pulse discriminating system for electrical discharge machining*, International Journal of Machine Tools and Manufacture, Vol.37, 511-522.

[29] Xue, J. (1985). *On multistate system analysis*, IEEE Transactions on Reliability, Vol.34, 329-337.

[30] Yeh, W. C. (2004). *Multistate network reliability evaluation under the maintenance cost constraint*, International Journal of Production Economics, Vol.88, 73-83.

[31] Yeh, W. C. (2011). *An improved method for multistate flow network reliability with unreliable nodes and a budget constraint based on path set*, IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans, Vol.41, 350-355.

[32] Zadeh, L. A. (1965). *Fuzzy sets*, Informat. Control, Vol.8, 338-353.

[33] Zhang, J. and Deng, Y. (2017). *A method to determine basic probability assignment in the open world and its application in data fusion and classification*, Applied Intelligence, Vol.46, 934-951.

[34] Zhang, Y. J., You-jun, L. I. and Shao-jun, L. I. (2014). *Study of boundaries of membership function values for slope fuzzy reliability analysis*, Rock and Soil Mechanics, Vol.35, 1157-1163.

[35] Zuo, M. J., Tian, Z. and Huang, H. Z. (2007). *An efficient method for reliability evaluation of multistate networks given all minimal path vectors*, IIE Transactions, Vol.39, 811-817.

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